Optical Rogue Waves in Integrable Turbulence

CEMPI CENTRE EUROPÉEN POUR LES MATHÉMATIQUES, LA PHYSIQUE ET LEURS INTERACTIONS

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Collaborators :

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Oceanography : M. Onorato (Turin, Italie)

Integrable Systems : G. El (Loughbourough, UK)

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OPTIROC

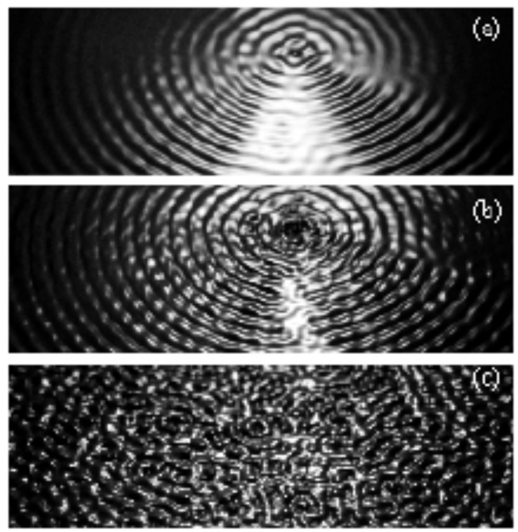
1. Context

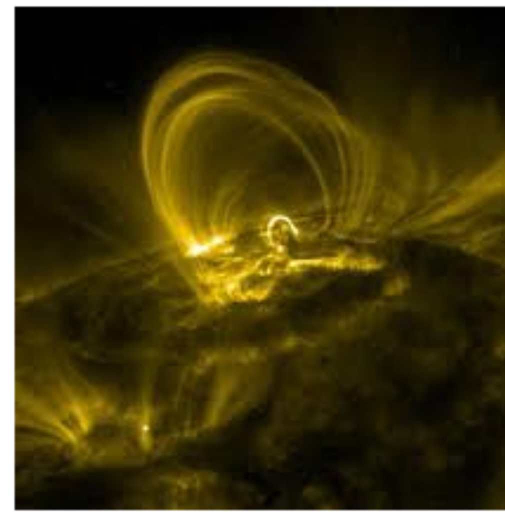
2. Experiments

3. Theoretical approach

Wave Turbulence

- ✓ **Waves**
  ubiquitous phenomena
  in Physics
- ✓ **Dispersion**
- ✓ **Nonlinearity**
- ✓ Often, **random**ness
  plays a significant role



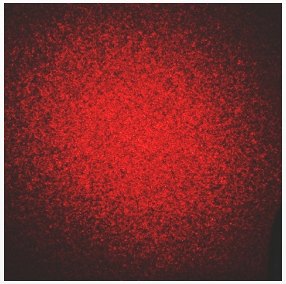


Wave turbulence = nonlinear dispersive random waves

hydrodynamics, oceanography, optics, mechanics, plasma physics…

Nonlinear Statistical Optics

Statistical Optics (linear)



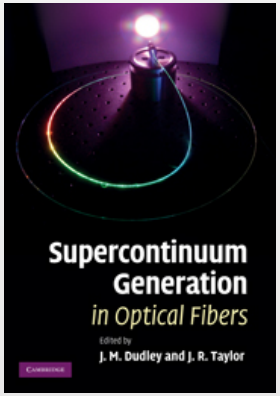
✓ XXth century Random / partially coherent light
(spatial or temporal fluctuation, coherence / spectrum…)

Nonlinear Statistical (fiber) Optics

✓ 1960s Lasers / nonlinear optics

✓ 1990s Supercontinuum
(ps, cw) : nonlinear + incoherent

A.Kudlinski and A. Mussot, Opt. Lett., **33**, (2008)
J. M. Dudley et al., Opt. Express **17**, (2009)



✓ 2000s Challenge : statistical description of nonlinear random waves
A. Picozzi *et al.*,
Optical wave turbulence : Toward a unified nonequilibrium thermodynamic formulation of statistical nonlinear optics
Physics Reports, (2014)

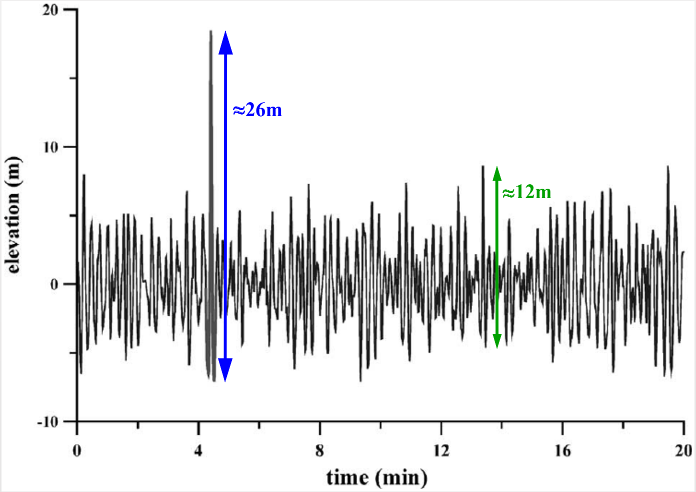
“Turbulence” in Optical fibers

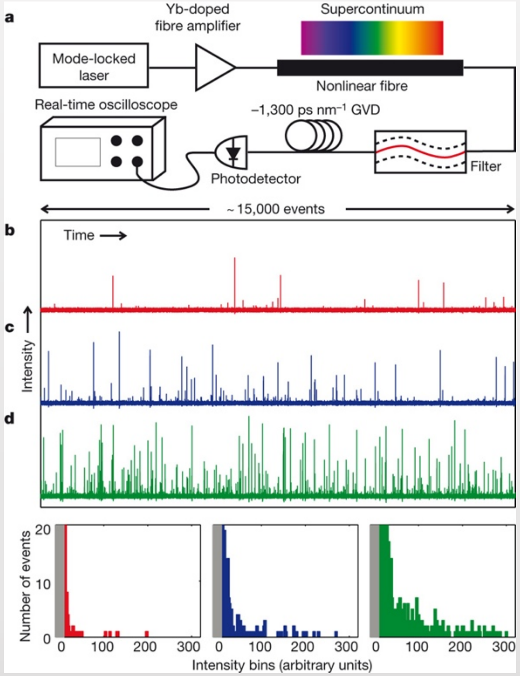
Optical fibers :
tabletop laboratories of hydrodynamical phenomena

✓ **Turbulence**

E.G. Turitsyna *et al.*, « The laminar–turbulent transition in a fibre laser », Nat. Photonics, **7** (2013)

✓ **Rogue waves**





D. R. Solli *et al.*, « Optical rogue waves », Nature **450**, 1054-1057 (2007)

The 1D Nonlinear Schrödinger Equation (1DNLSE)

$$\mathrm{i}\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} - \gamma|\psi|^2\psi$$

✓ **Spatio-Temporal dynamics at leading order of numerous systems**

BEC, nonlinear optics, water waves…
Narrow band approximation

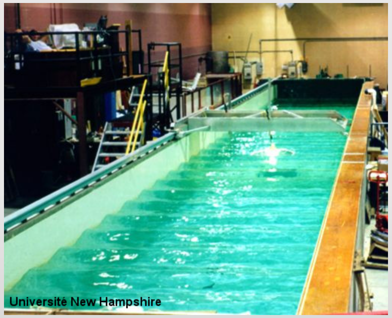
✓ **Deep water waves (focusing) / Single mode fiber**

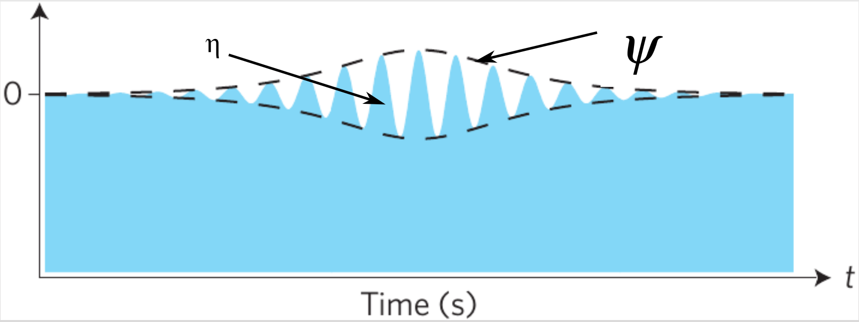
water wave elevation

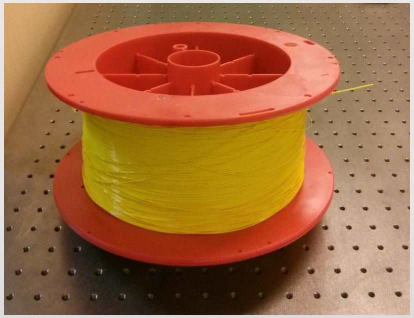
$$\eta = \Re(\psi\, e^{i(k_0 z - \omega_0 t)})$$

electric field

$$E(x,y,z,t) = \Re\big(A(x,y)\ \psi(z,t)\ e^{i(k_0 z - \omega_0 t)}\big)$$







J. Dudley *et al.*, Nat. Photonics 8, 755, (2014)   A. Chabchoub *et al.*, Annals of Physics 361, 490, (2015)

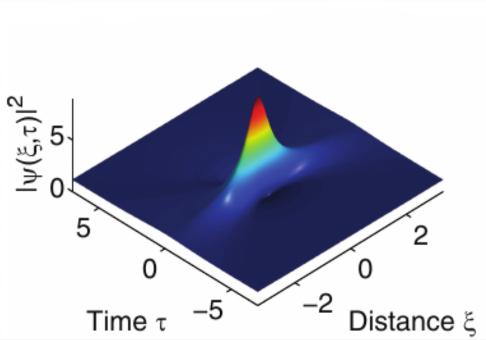
1D-NLSE is integrable

- ✓ **Inverse scattering transform**

- ✓ **Exact solutions**
  - ➢ Solitons
  - ➢ Solitons on finite background (SFB)

    Peregrine solitons, Akhmediev breathers…
  - ➢ SFB : proptotype of rogue waves ?

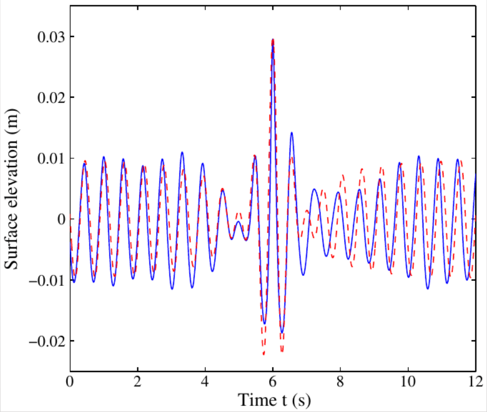
    N. Akhmediev *et al.*, Physics Letters A, 373, 2009



- ✓ **Experiments with *coherent* initial conditions**
  - ➢ Optical fibers

    B. Kibler *et al.*, Nature Physics **6**, 790, (2010)
    B. Kibler *et al.*, Scientific Reports, **2**, 790, (2012)
    B. Frisquet *et al.*, Phys. Rev. X, **3**, 041032, (2013)
  - ➢ Water tank

    Chabchoub *et al.*, Phys. Rev. Lett. **106**, 204502, (2011)
    B. Kibler *et al.*, Phys. Rev. X, **5**, 041026 (2015)



Integrable Turbulence

✓ **Random initial conditions + integrable system (1D-NLSE)**

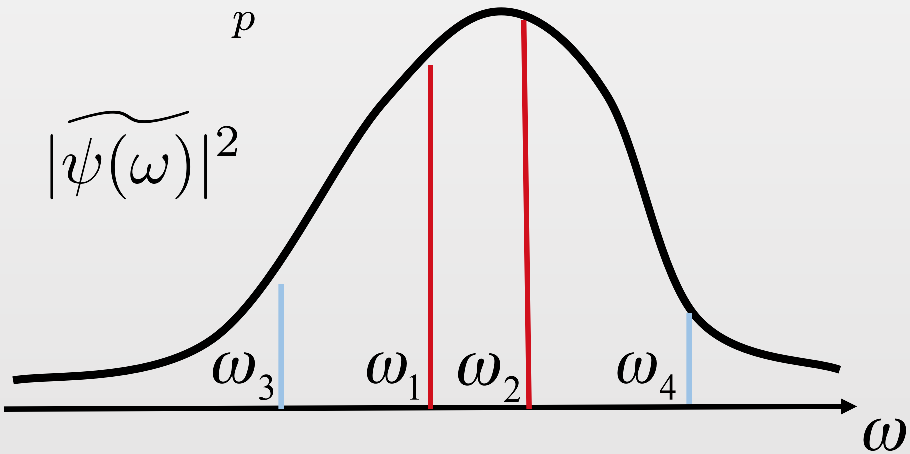
V.E. Zakharov, Turbulence in Integrable Systems, 2009

D.S. Agafontsev and V.E. Zakharov, Integrable turbulence and formation of rogue waves, Nonlinearity, 2015

J. Soto-Crespo et al., Integrable Turbulence and Rogue Waves : Breathers or Solitons ?, Phys. Rev. Lett., 2016

✓ **Non resonant four waves mixing (FWM)**

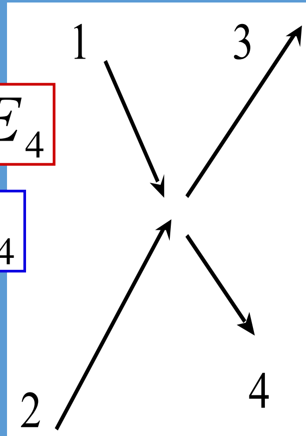
$$\psi(t) = \sum_p \widetilde{\psi(\omega_p)}\, e^{i\omega_p t}$$



Collisions in gas

$$E_1 + E_2 = E_3 + E_4$$

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4$$



$$k(\omega_1) + k(\omega_2) = k(\omega_3) + k(\omega_4)$$

$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

Integrable Turbulence

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D.S. Agafontsev and V.E. Zakharov, Integrable turbulence and formation of rogue waves, Nonlinearity, 2015

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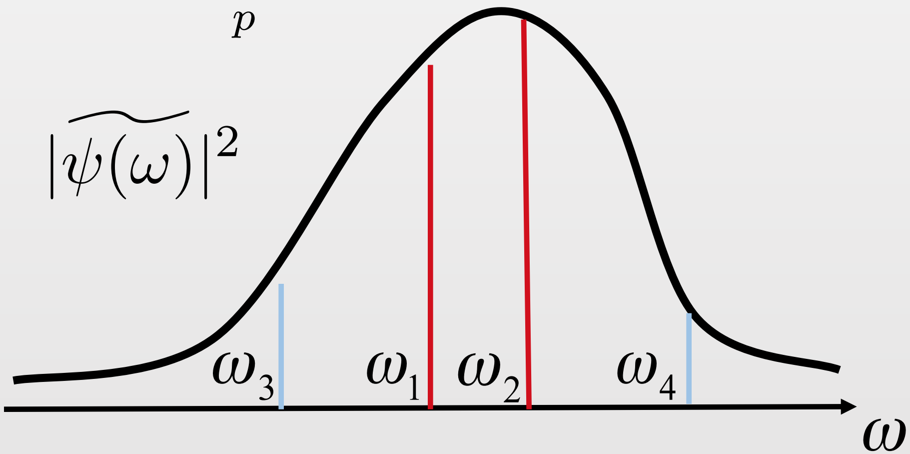

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$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

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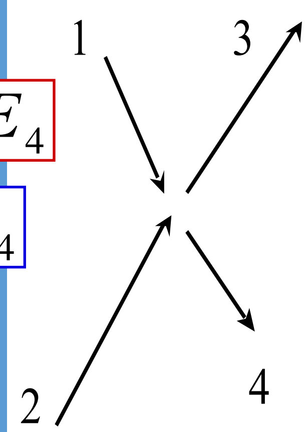
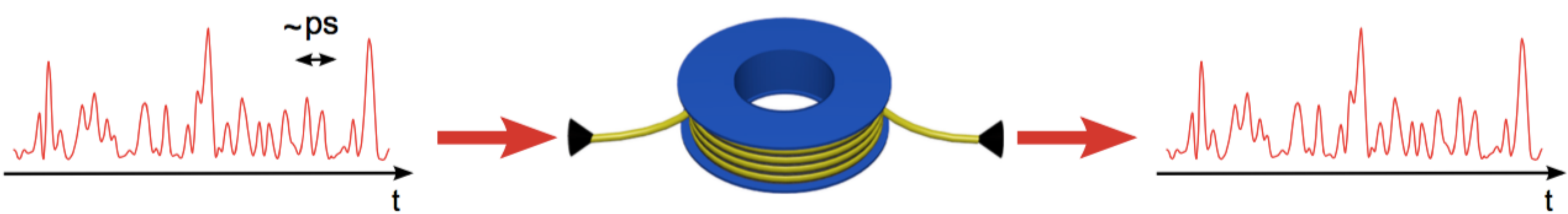


$$\mathrm{i}\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} - \gamma|\psi|^2\psi$$

$$k(\omega) = \frac{\beta_2}{2}\omega^2$$

Principle of experiments



Initial partially coherent waves

- ✓ Linear superposition of independent waves $$\psi(t) = \sum_p \widetilde{\psi(\omega_p)}\, e^{i\omega_p t}$$
- ✓ Probability Density Functions (PDF)
  - ➢ Field : Gaussian statistics (central limit theorem)

$$PDF\left[\Re(\psi)\right] = \exp\left[-\Re(\psi)^2\right]$$

  - ➢ Power : exponential

$$PDF(P/<P>) = \exp(-P/<P>)$$

Nonlinear propagation described by
The focusing 1D-NLSE

$$i\frac{\partial\psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2\psi}{\partial t^2} - \gamma|\psi|^2\psi$$

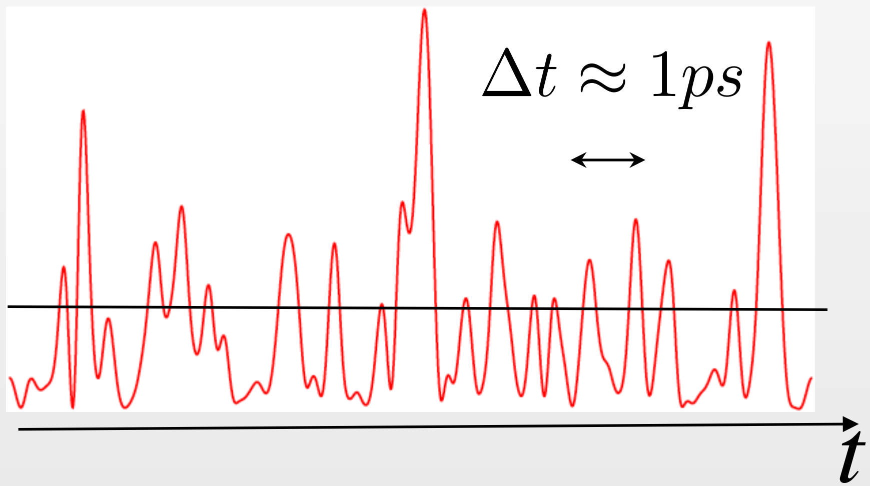
2. Experiments
Fast measurement

Statistics (Optical Sampling)

Dynamics (Time lens – Time Microscope)

Experimental challenges

- ✓ Time scale <1ps



- ✓ Standard photodetectors / Oscilloscope Δt<20 ps
- ✓ Irregular / random evolution (non reproductible)

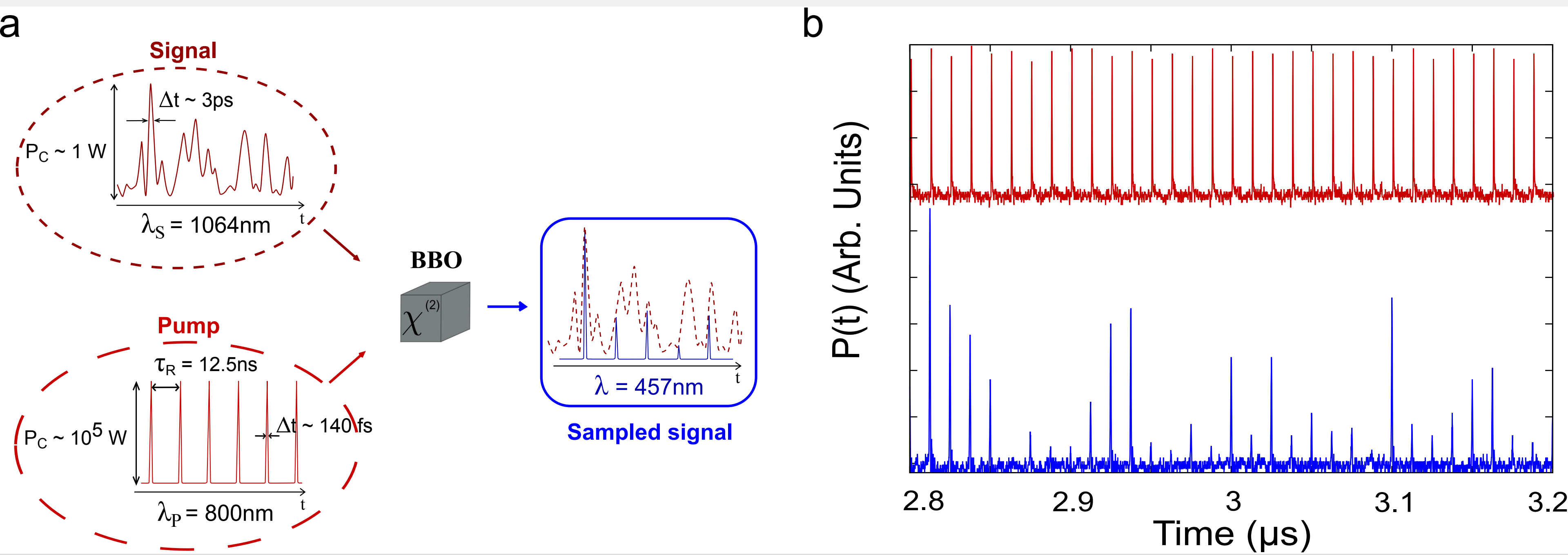
Fast measurement of statistics (random light)

✓ **Asynchronous Optical Sampling**

Sum frequency generation signal + fs pulses

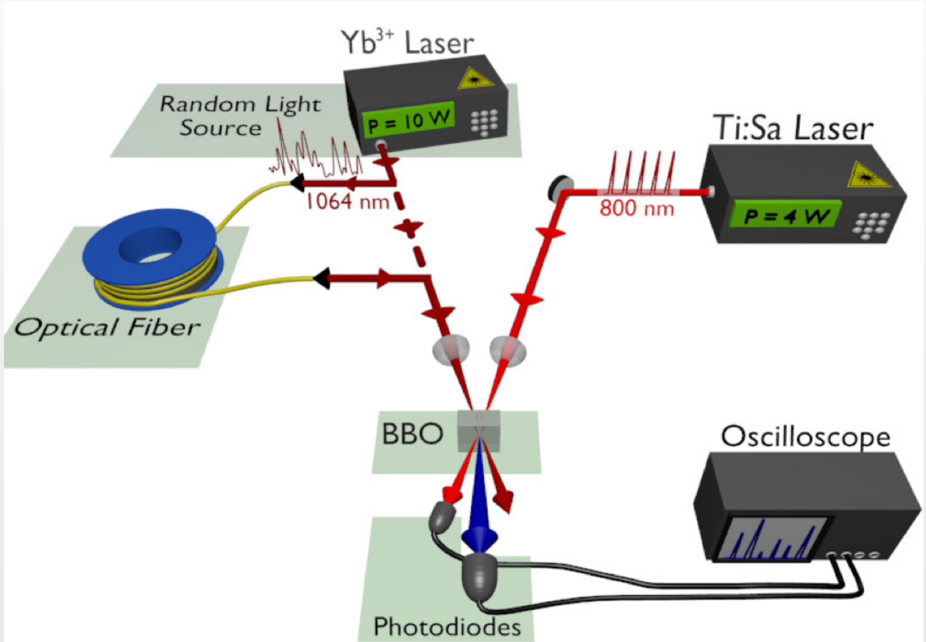
Temporal resolution : 250 fs

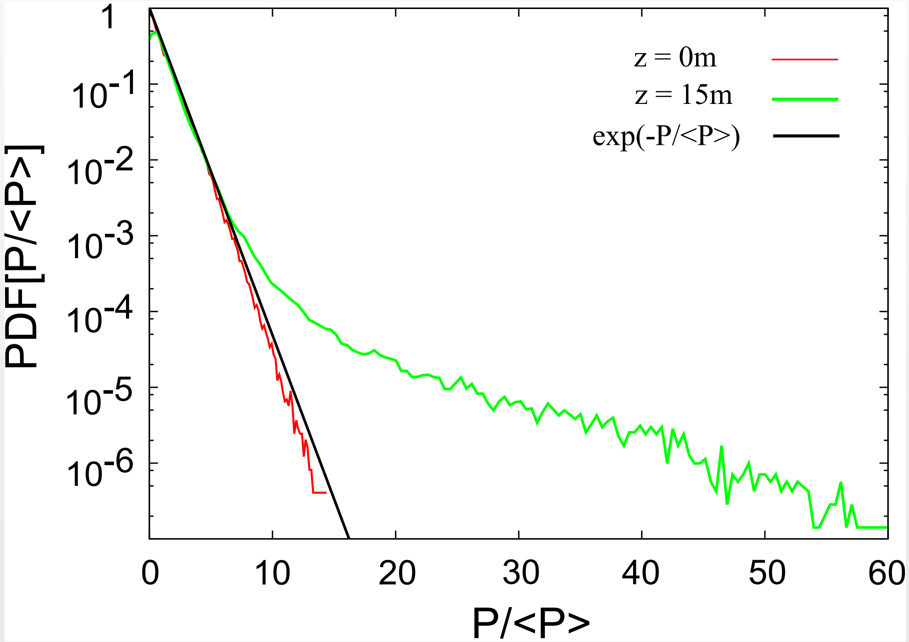
P. Walczak *et al.*, Phys. Rev. Lett., 2015



Fast measurement of statistics

Strong Deviation from the Gaussian statistics





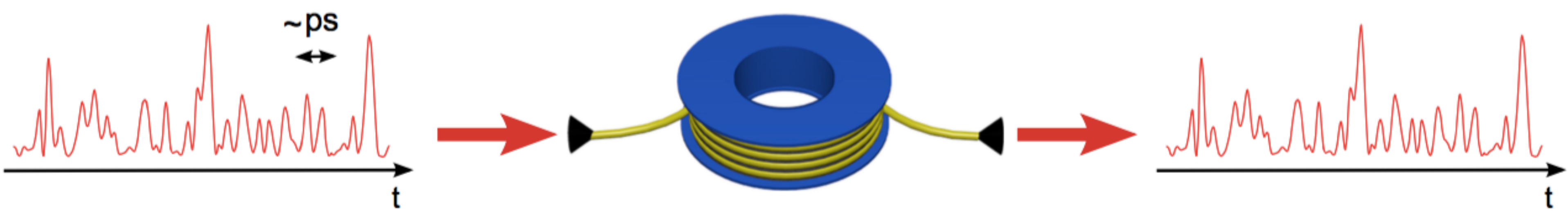
Occurrence of P=50<P> ?

1 / 10^{10} seconds @ z=0m

1 / 10^{-6} seconds @ z=15m

P. Walczak *et al*., Phys. Rev. Lett., 2015

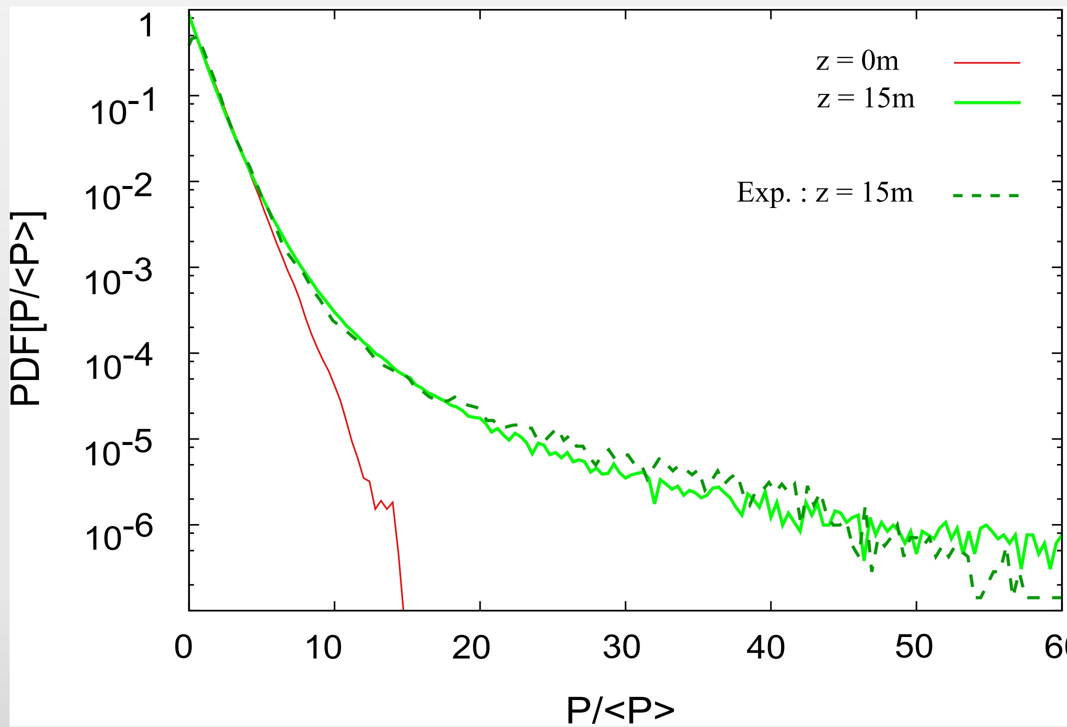
Numerical simulations



Emergence of strongly non Gaussian statistics

$$\psi(t) = \sum_p \widetilde{\psi(\omega_p)}\, e^{i\omega_p t}$$

$$\mathrm{i}\frac{\partial\psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2\psi}{\partial t^2} - \gamma|\psi|^2\psi$$

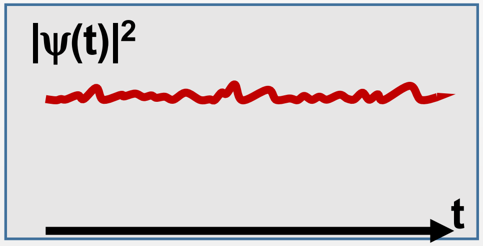


P. Walczak *et al.*, Phys. Rev. Lett., 2015

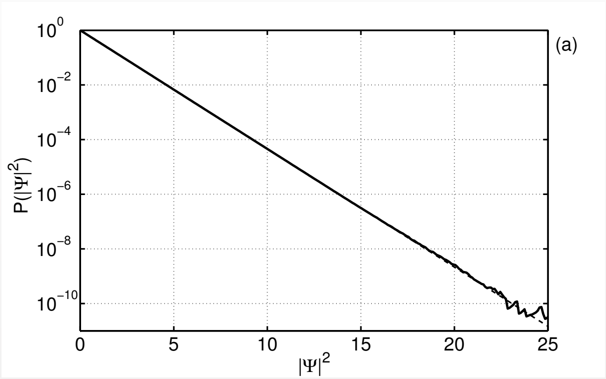
Influence of the initial condition

✓ **Noise driven modulational instability**

D.S. Agafontsev and V.E. Zakharov, Nonlinearity, 2015

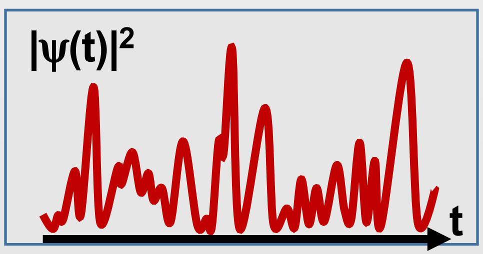


Stationnary state :
Gaussian statistics

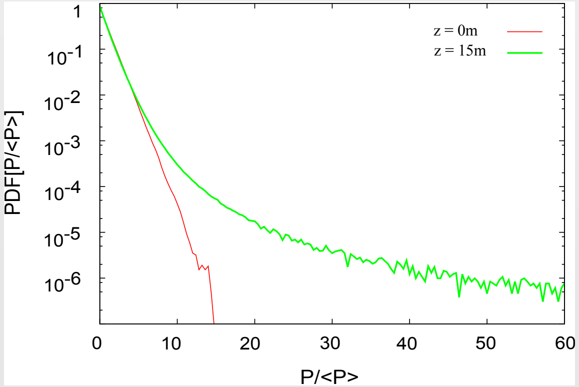


✓ **Strongly fluctuating initial condition**

P. Walczak *et al.*, Phys. Rev. Lett., 2015



Stationnary state :
strongly non Gaussian statistics

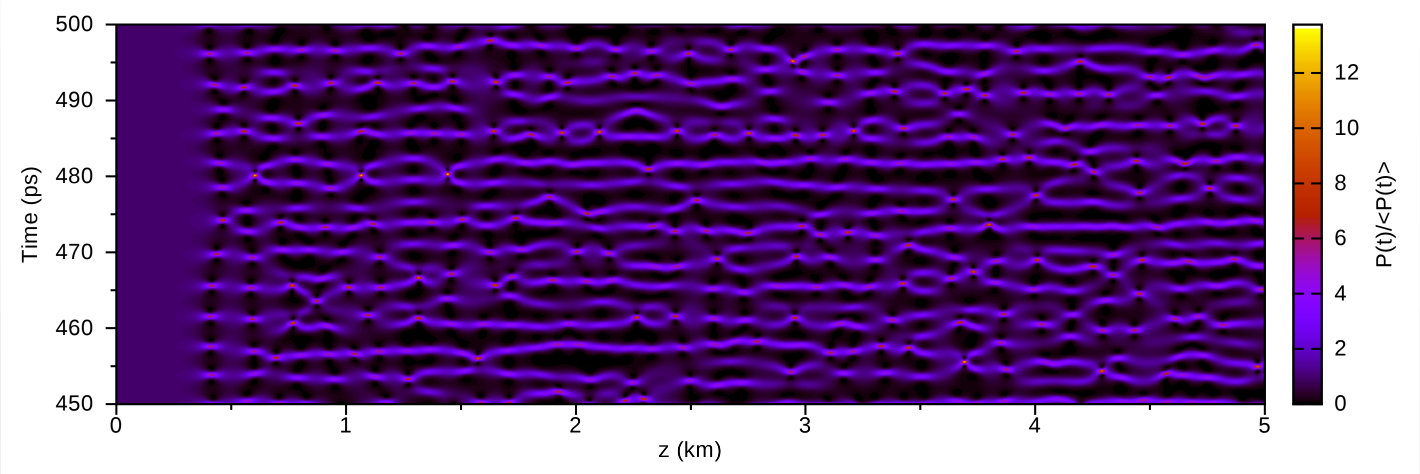


Open theoretical question : transition between the two cases

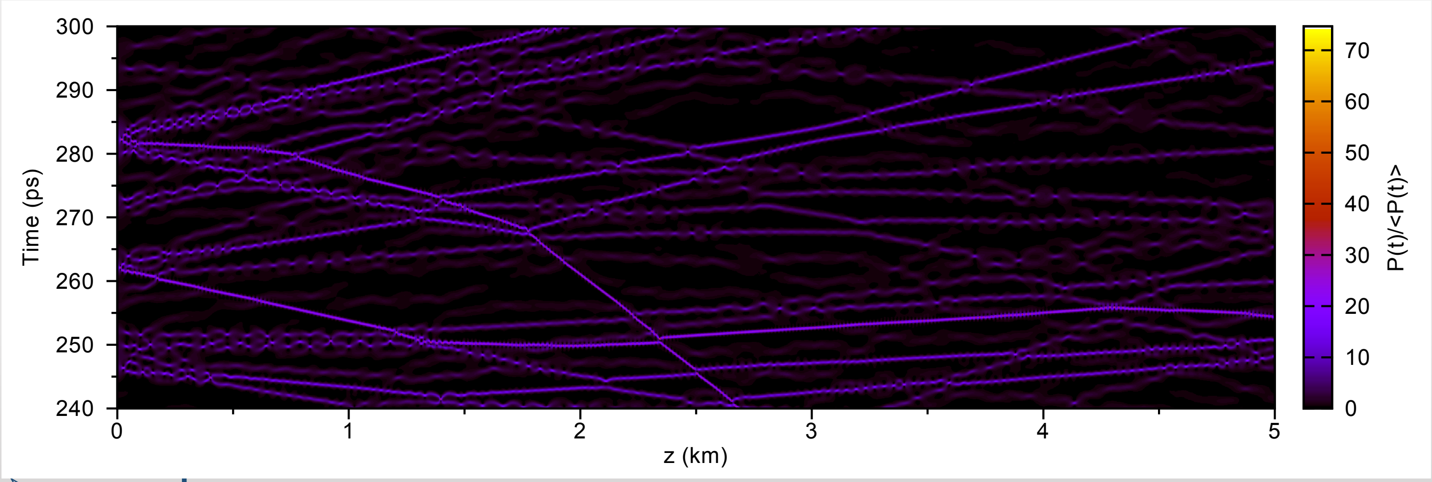
J. Soto-Crespo et al., Integrable Turbulence and Rogue Waves : Breathers or Solitons ?, Phys. Rev. Lett., 2016

Influence of the initial condition

- ✓ **Noise driven modulational instability** Dudley et al. Nat. Photon. 8, 75 (2014)



- ✓ **Strongly fluctuating initial condition**



- ➢ movie

Observation of the fast dynamics

- ✓ **Single shot recording**
  (irregular, non reproductible fluctuations)

- ✓ **"Large" temporal window**
  25-30 ps

- ✓ **Ultrafast measurement**
  resolution 250fs

- ✓ **Strategy : Time Microscope (1 Time lens + Spectral observation)**

Collaboration : C. Evain, C. Szwaj, S. Bielawski (Phlam)

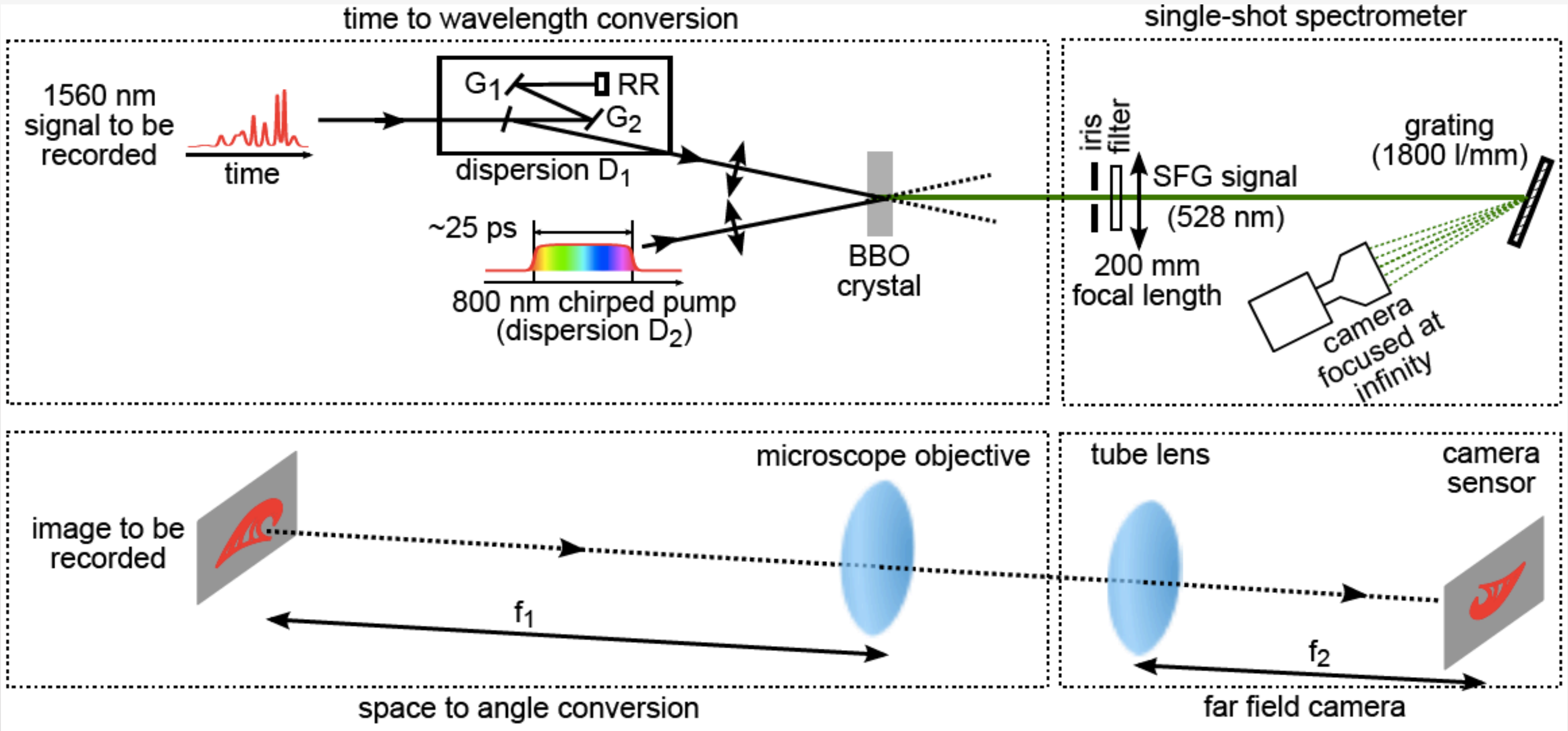
Kolner et al., Opt. Lett. 14, 630 (1989)
Bennett and Kolner, Opt. Lett. 24, 783 (1999)
Foster et al., Nature 456, 81 (2008)

First single shot observation of optical rogue waves

P. Suret et al., Nature Communications (2016)

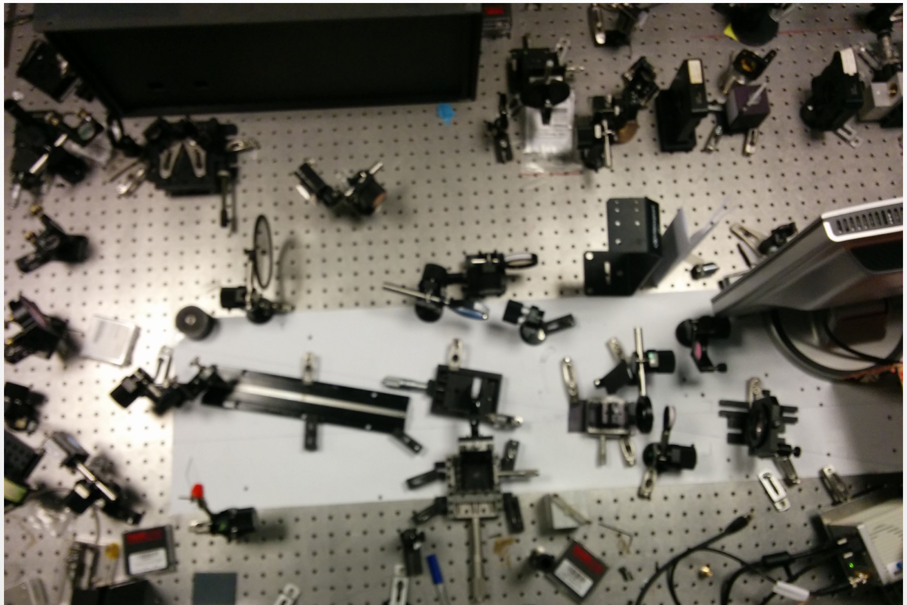
Time Microscope

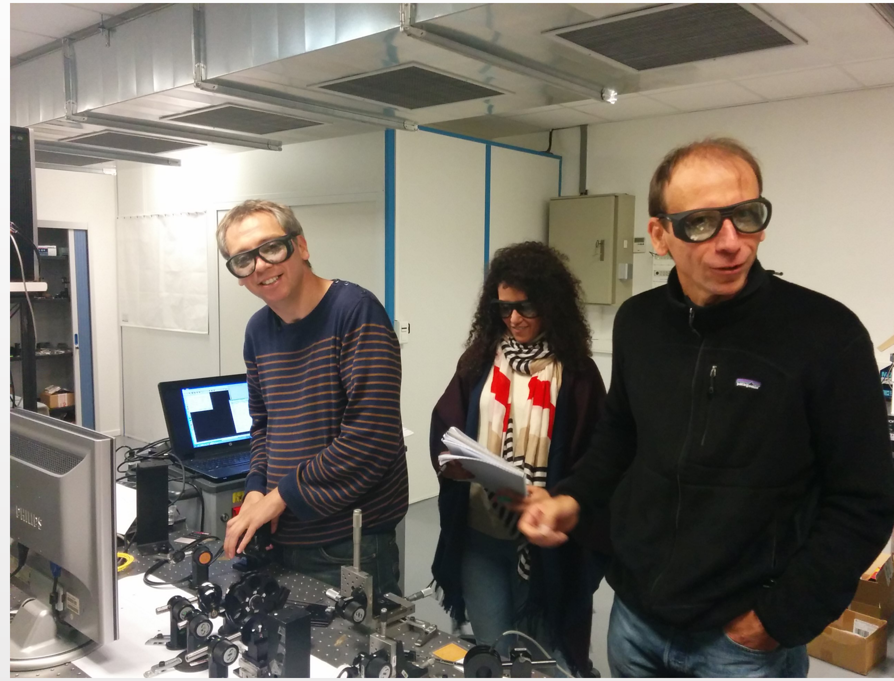
- ✓ **Equivalence Time / Space**
  - ➢ Dispersion / Diffraction
  - ➢ Lens : Quadratic phase
  - ➢ Optical spectrum / far field

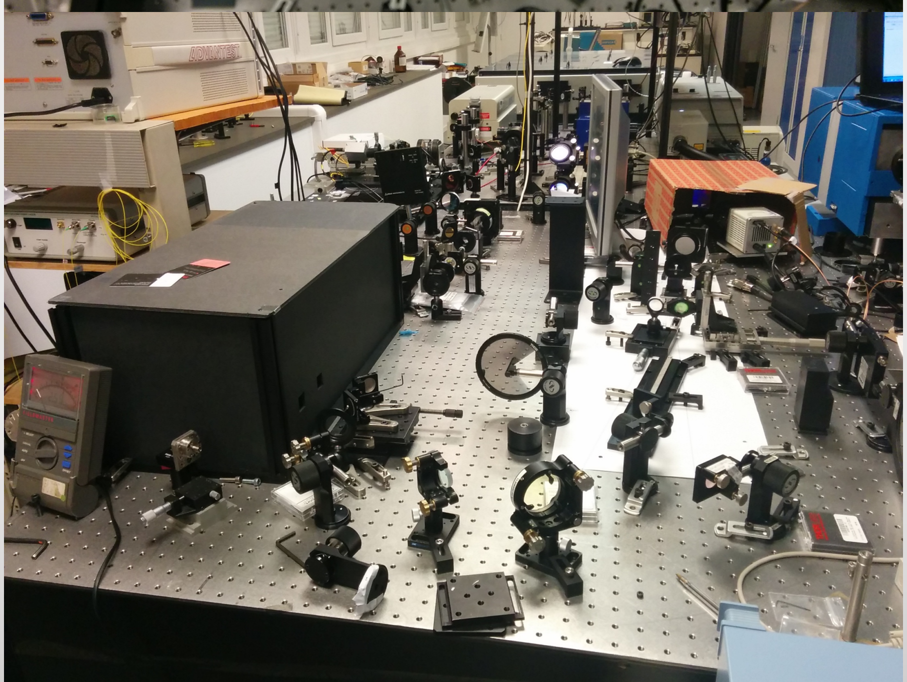


- ✓ **Characteristics : single shot / 250 fs / 30 ps / 500Hz**

Time Microscope





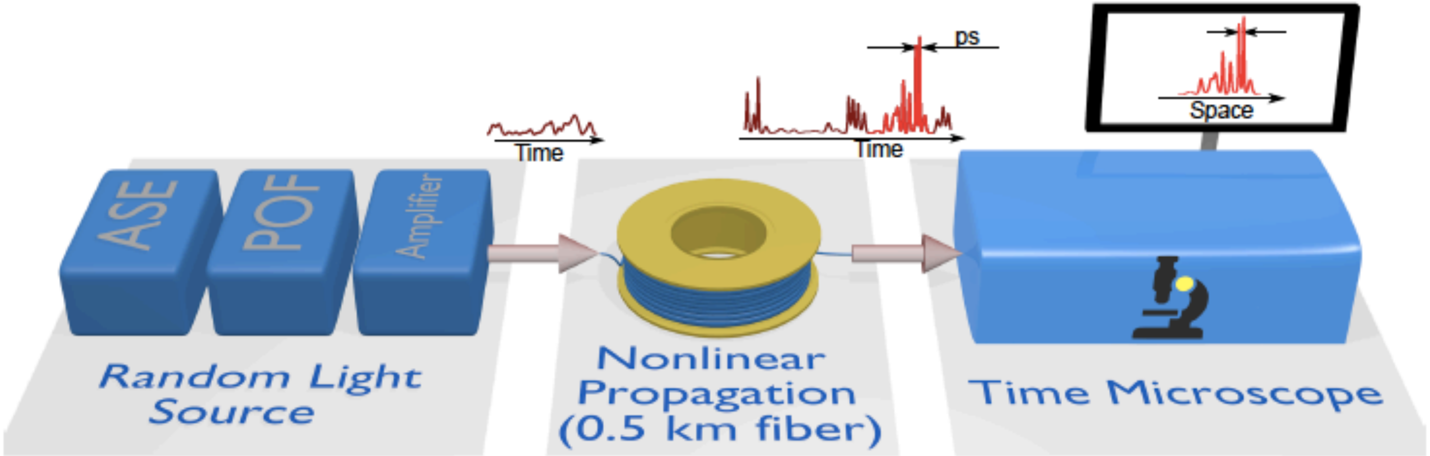


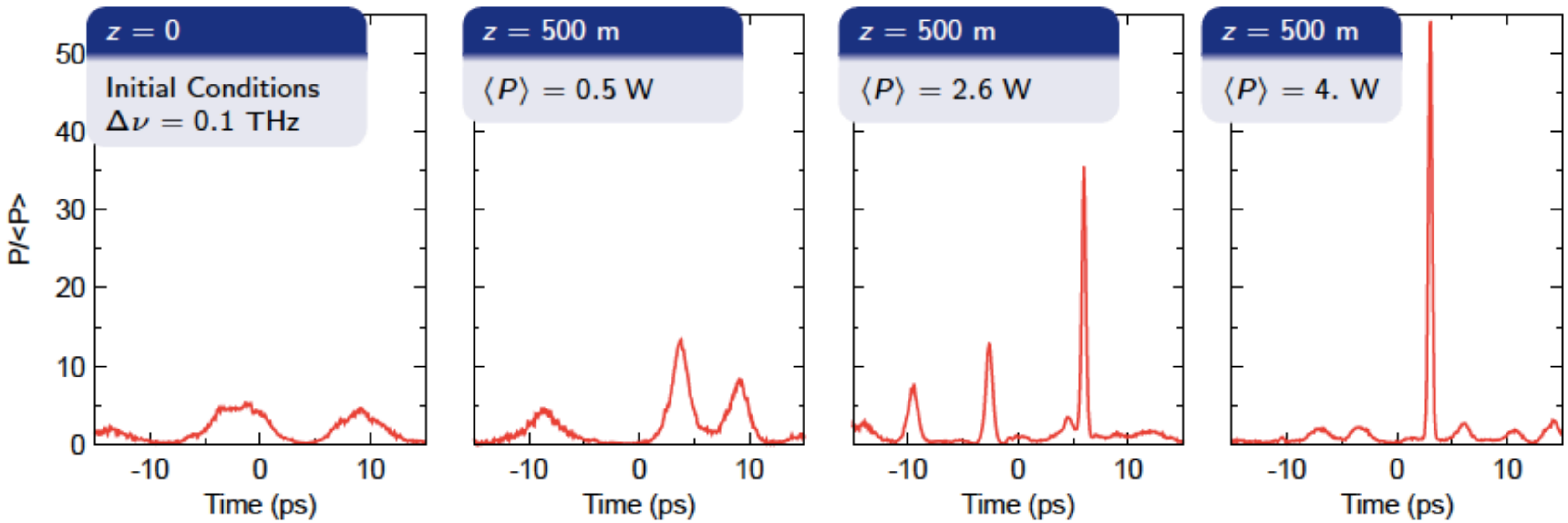
Time Microscope

Nonlinear propagation of partially coherent Waves

Spontaneous Emission + progammable optical filter (POF)

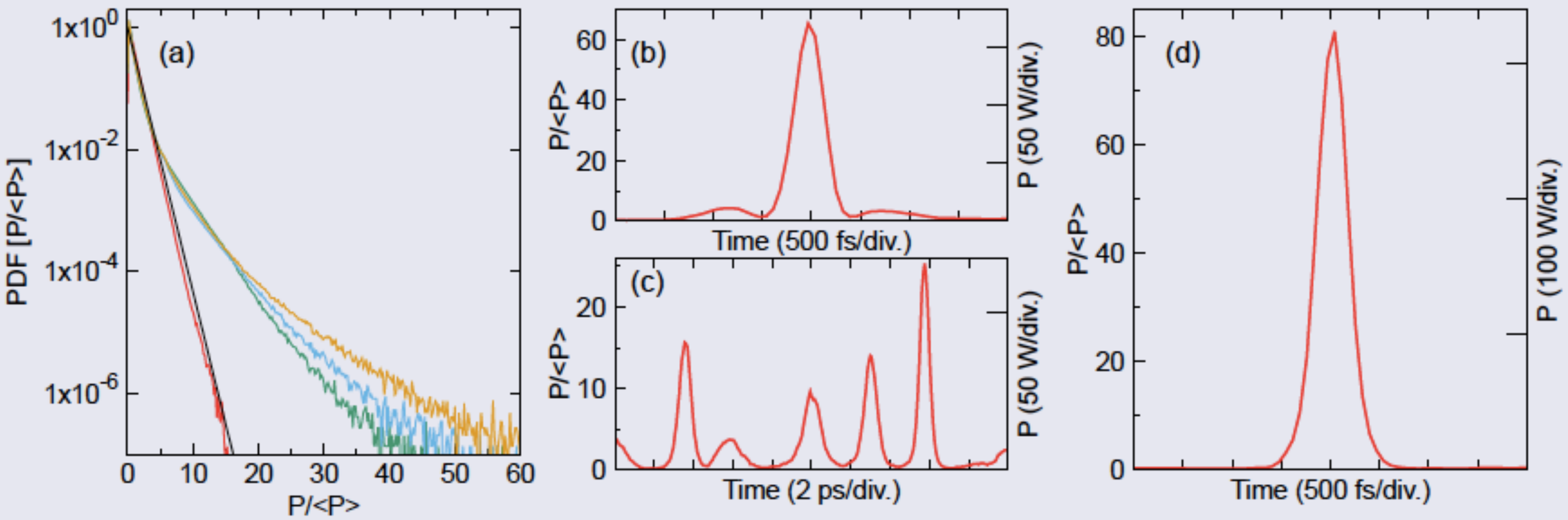
Tunable spectral width $\Delta\nu = 0.1$ THz



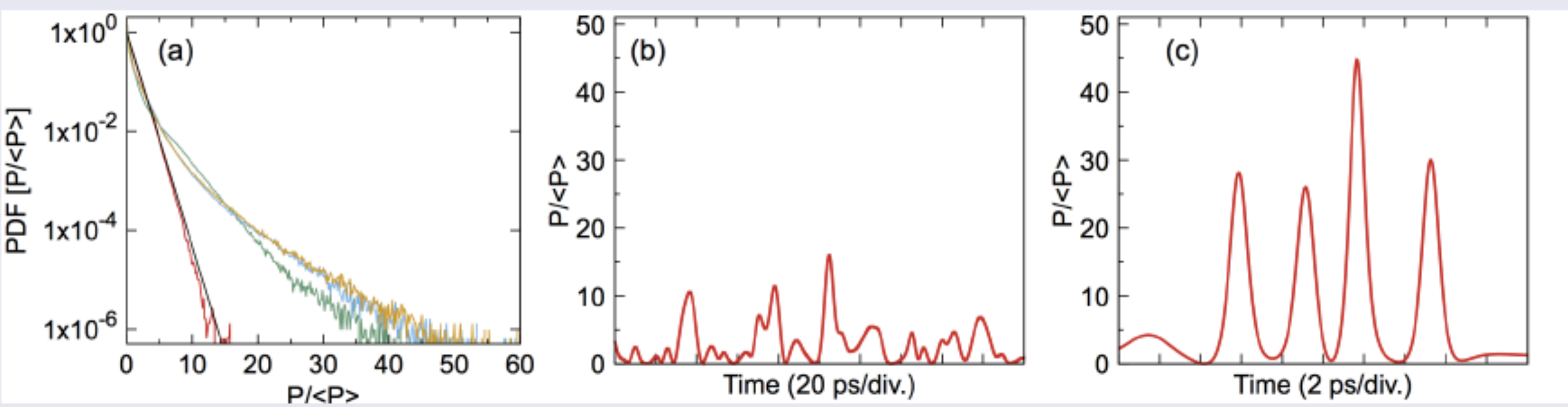


Time Microscope

Experiments

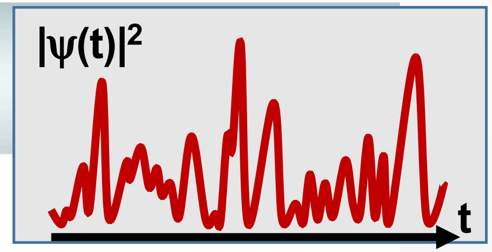


Numerical Simulations of 1D-NLSE



Influence of the initial condition

nature COMMUNICATIONS



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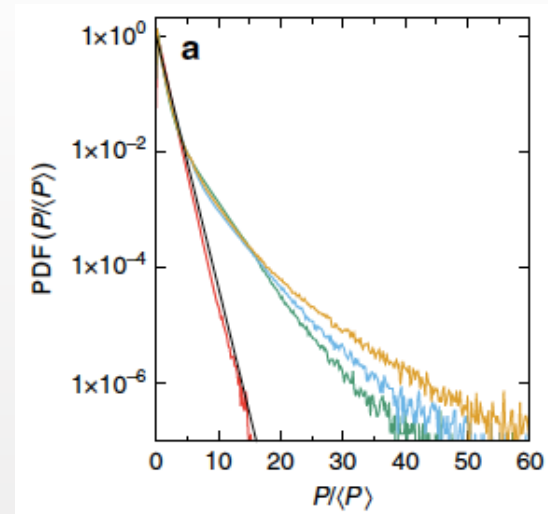
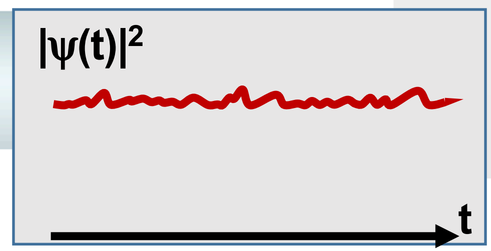
Single-shot observation of optical rogue waves in integrable turbulence using time microscopy

Pierre Suret[1,2], Rebecca El Koussaifi[1,2], Alexey Tikan[1,2], Clément Evain[1,2], Stéphane Randoux[1,2], Christophe Szwaj[1,2] & Serge Bielawski[1,2]



nature COMMUNICATIONS



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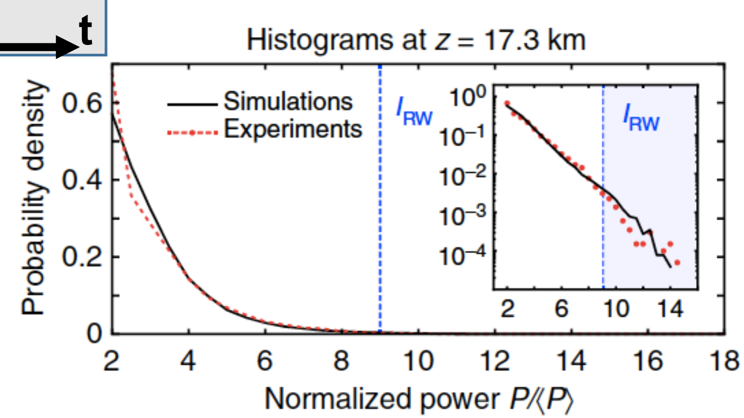
Received 8 Apr 2016 | Accepted 25 Oct 2016 | Published 19 Dec 2016 DOI: 10.1038/ncomms13675 OPEN

Real-time measurements of spontaneous breathers and rogue wave events in optical fibre modulation instability

Mikko Närhi[1], Benjamin Wetzel[2,3], Cyril Billet[4], Shanti Toenger[1,4], Thibaut Sylvestre[4], Jean-Marc Merolla[4], Roberto Morandotti[2,5,6], Frederic Dias[7], Goëry Genty[1] & John M. Dudley[4]



3. Theoretical approaches

Wave Turbulence Theory

Exact Relation

Inverse Scattering Transform

Semi-Classical approach

Toward a theory of Integrable turbulence ?

- ✓ Wave turbulence theory (kinetic theory)
  - ➢ Weakly nonlinear regime

    A. Picozzi *et al.*, Physics Reports, (2014)

$$\left\langle \tilde{\psi}(z,\omega)\, \tilde{\psi}^*(z,\omega') \right\rangle = n_\omega(z)\, \delta(\omega - \omega')$$

- ✓ Exact relation between spectrum and statistics

  M. Onorato, *et al.* Phys. Lett. A, **380** , 39, (2016)

$$\kappa = \frac{\langle |A|^4 \rangle}{\langle |A|^2 \rangle^2}$$

$$\kappa(x) = \kappa(x_0) + 2\frac{\beta}{\alpha}\frac{1}{\langle N \rangle}\left[\Omega(x)^2 - \Omega(x_0)^2\right]$$

- ✓ Semiclassical limit of 1D-NLSE
  - ➢ Strongly nonlinear regime

$$i\epsilon\frac{\partial \psi}{\partial z} + \epsilon^2 \frac{\partial^2 \psi}{\partial t^2} + |\psi|^2 \psi = 0$$

$$\psi = \sqrt{\rho} e^{i\phi/\epsilon}$$

- ➢ Collaboration with Gennady El
- ➢ Hydrodynamical formulation of nonlinear optics
- ➢ Random Riemann waves

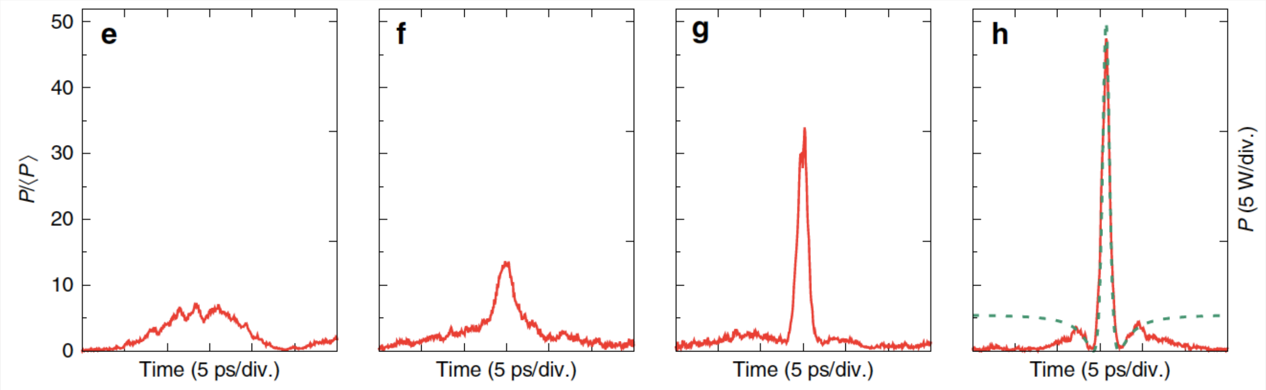
  S. Randoux et al., arXiv:1702.00006, 2017

Semiclassical limit of 1D-NLSE

✓ Universal mechanism : Peregrine soliton

M. Bertola and A. Tovbis , "Universality for the Focusing Nonlinear Schrödinger Equation at the Gradient Catastrophe Point : Rational Breathers and Poles of the Tritronquée Solution to Painlevé I", Comm. on Pure and Applied Mathematics, (2013)

✓ Ingredient of integrable turbulence



✓ Collaboration with J. Dudley, G. Genty *et al.*

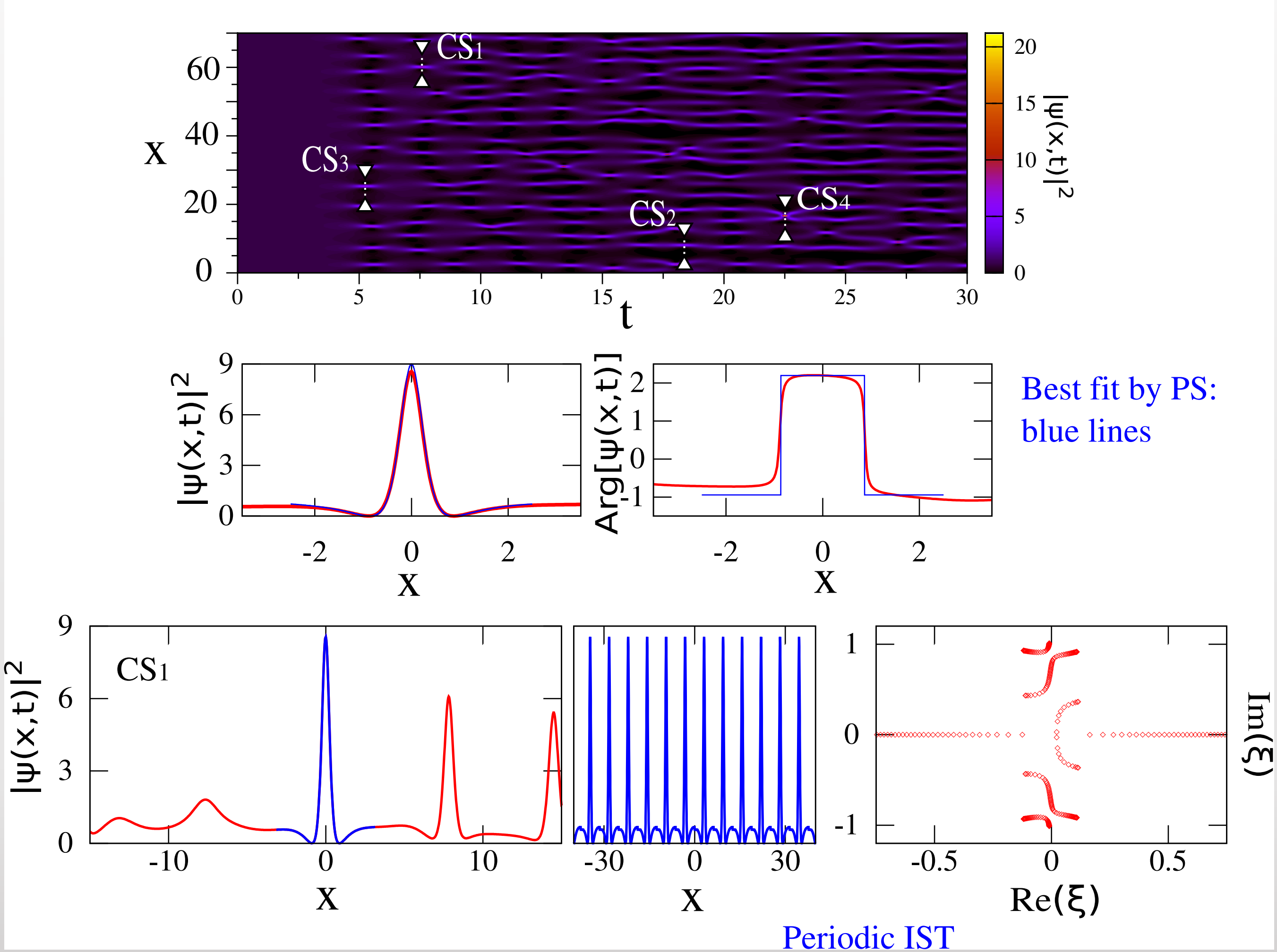
- ➢ Pulse propagation / Peregrine soliton
- ➢ Measurement of Intensity and phase

A. Tikan et al., arXiv:1701.08527, 2017

Inverse scattering transform

- ✓ Nonlinear analysis of local structures emerging in integrable turbulence

S. Randoux, P. Suret, G. El, Scientific Report 6, 29238 (2016)
“Inverse scattering transform analysis of rogue waves using local periodization procedure,”



Conclusion

- ✓ Front edge fast measurement techniques in optics
  - ➢ Time lens / time microscope
- ✓ Open new fundamental and theoretical questions
  - ➢ rogue waves / integrable turbulence
  - ➢ Influence of initial statistics
  - ➢ Role of non resonant interaction in turbulence
- ✓ Perspectives
  - ➢ Application of fast measurement (pulsed lasers, ultrafast imaging…)
  - ➢ New mathematical problems
    - Integrable / non integrable (interdisciplinary CNRS project - CEMPI)
    - Inverse Scattering Transform / perdiodic boundary conditions

Conclusion

Articles

[1] S. Randoux , P. Walczak, M. Onorato and P. Suret, **Phys. Rev. Lett.**, 113, 113902, (2014)
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[5] M. Onorato and P. Suret, **Natural Hazards**, **84** , 541-548, (2016)
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Chapters in book

Rogue and Shock Waves in Nonlinear Dispersive Media
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[1] Hydrodynamic and Optical Waves : A Common Approach for Unidimensional Propagation
M. Onorato, F. Baronio, M. Conforti, A. Chabchoub, P. Suret and S. Randoux
[2] Integrable Turbulence with Nonlinear Random Optical Waves
S. Randoux and P. Suret

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