Mini cours :
Approximation and extremal problems
du Pr. William Ross
(University of Richmond, Virginia, USA)

1 Calendrier

-Mardi 26 mai de 10h à 12h : salle de séminaire du M3
-Jeudi 28 mai de 14h à 16h : salle de séminaire du M3
-vendredi 29 mai de 10h à 12h : salle de séminaire du M3

2 Programme

Lecture 1 : Classical extremal problems from the birth of complex analysis

We begin with the classical Cauchy's formula on the disk and use this as a starting point to state some extremal problems for the Hardy space. We outline some historical results of Fejer, Egervary, Golusin, Khavinsin, and Shapiro. This first lecture is to outline the two goals of the project : computing the extremal problem and computing the extremal function.

Lecture 2 : Dual problems

Some of the main tools used to explore extremal problems come from functional analysis. By means of duality, we can covert our integral extremal problem from Lecture 1 into distance to the bounded analytic functions.

Lecture 3 : Towards a solution via model spaces

Our approach to extremal problems will be to convert them into problems involving the classical model spaces. For this we need to introduce inner functions, the Hilbert spaces Hardy space, and model spaces. These are perhaps new concepts for graduate students and so we start from scratch.

Lecture 4 : Operator theory comes to the rescue

Using model spaces, we can recast extremal problems into the language of norms of truncated Toeplitz operators on model spaces. These operators are part of a general class of operators called complex symmetric operators and it is the properties of these operators which allow us to give a more modern and computationally viable technique of solving extremal problems. We will also connect all of this to the theory of Hankel operators.

Lecture 5 : Quadratic extremal problems

Via operator theory, model spaces, and truncated Toeplitz operators, we can examine our linear extremal problem as an equivalent quadratic extremal problem. We will show that under certain certain circumstances, the extremal function for the linear extremal problem is the square of an outer function. The main tool here will be complex symmetric operators.

Lecture 6 : From here to H-infty

Here we examine a new type of extremal problem – a Fourier extremal problem — which connects in some tangential way to the circle method of Hardy and Littlewood used to address various number theory problems