Unveiling new topological phases with optical fiber networks

Albert Adiyatullin, Corentin Lechevalier, Rabih El Sokhen,
Clement Evain, François Copie, Stéphane Randoux,
Pierre Suret, Alberto Amo

In collaboration with:
Lavi Upreti (Univ. Würzburg), Pierre Delplace (Univ. Lyon)

CEMPI days • 31/03/2022

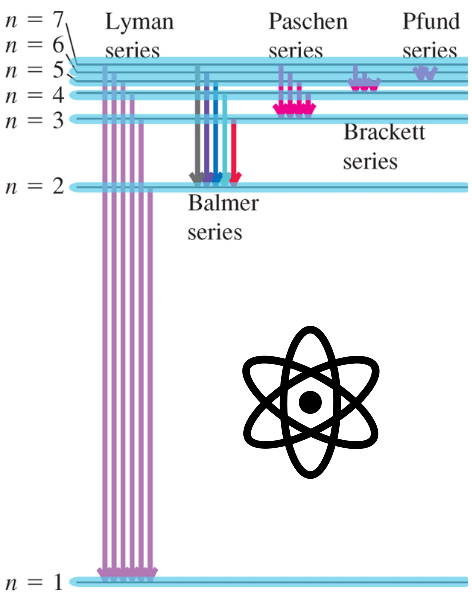
Plan

I. Introduction: topology, Floquet driving

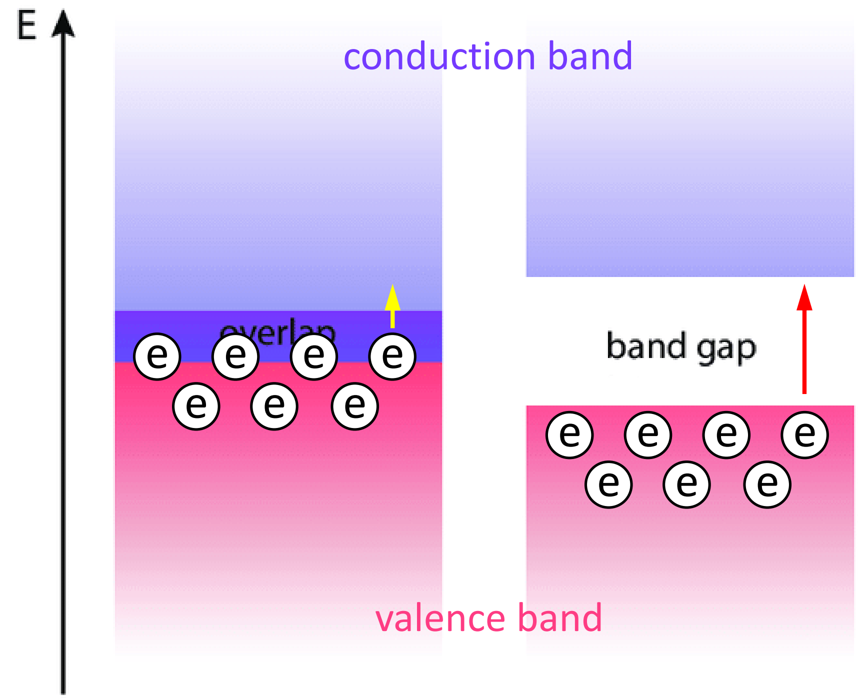
II. Programmable 2D photonic network

III. Floquet winding metals

Atom: energy *levels*



Crystal: energy *bands*



CONDUCTORS

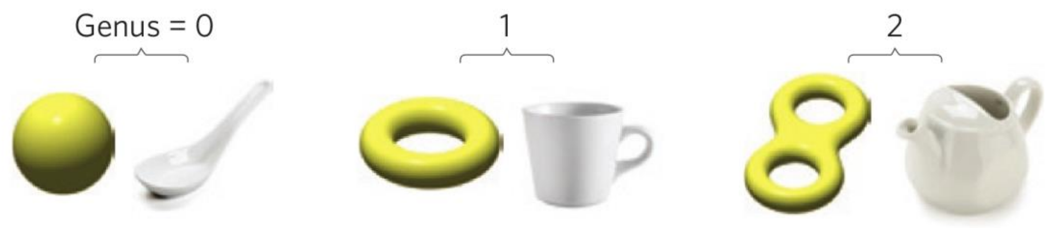
INSULATORS

incandescent bulb

LED lamp

Topological invariant

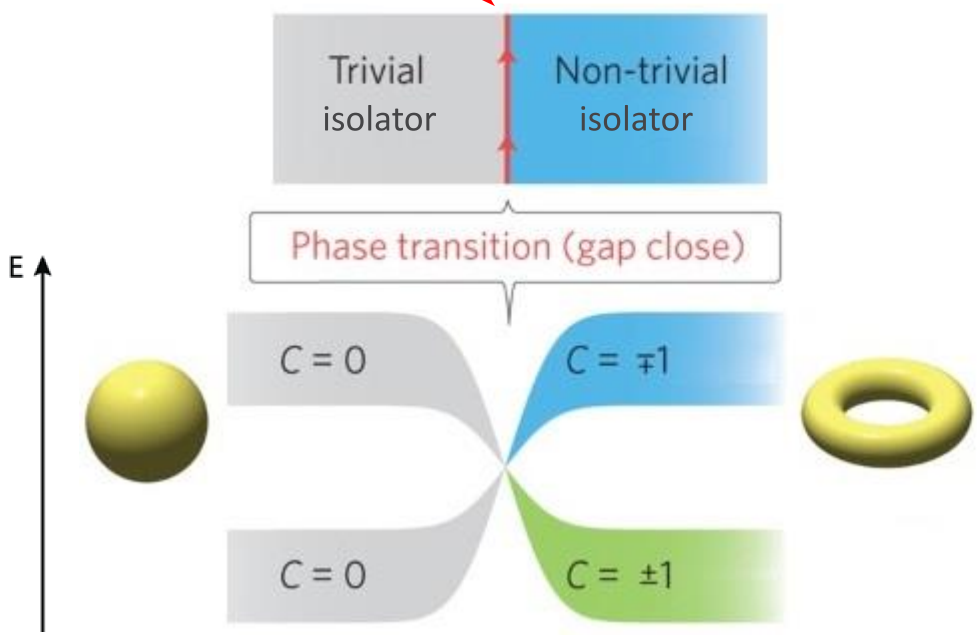
Mathematical example: number of holes



Lu et al., Nature Phot. 2014

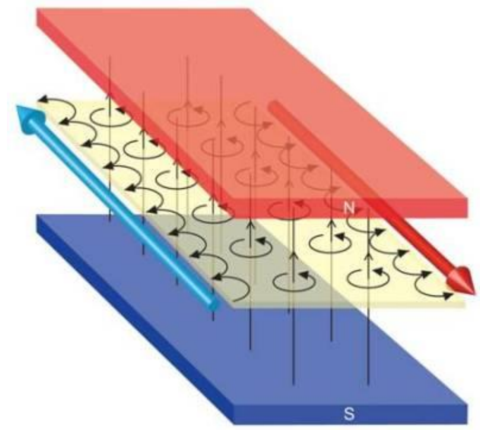
- integer number
- can be defined for each band
- immune to perturbations

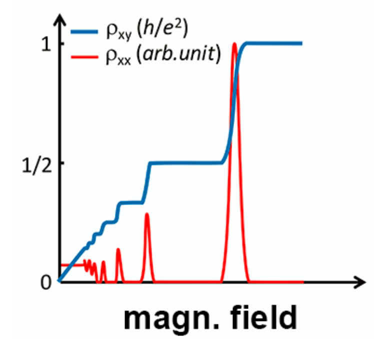
Invariant can change only if the gap closes conductive edge



Lu et al., Nature Phot. 2014

Quantum Hall effect

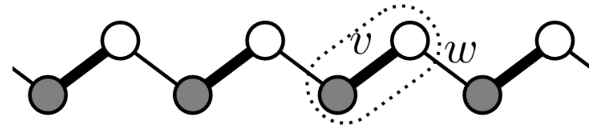




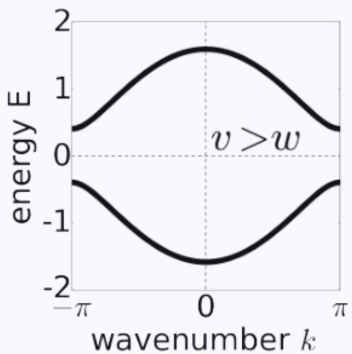
topological charge ↔ presence of edge states

An example

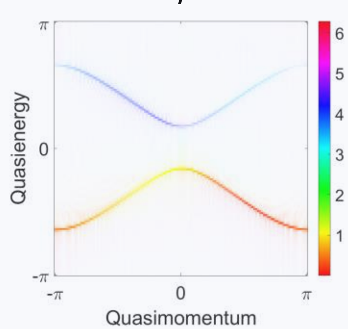
Bipartite lattice (2 sites per unit cell):



Trivial: $v > w$



ϕ

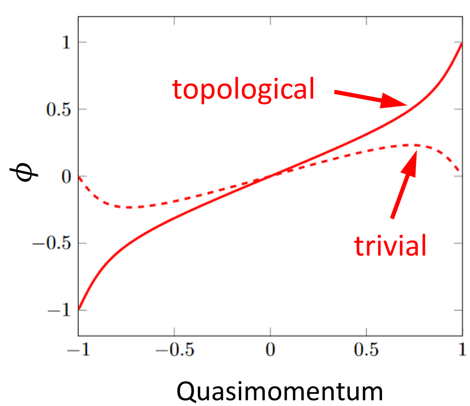


Stationary Schrödinger equation $\hat{H}\Psi = E\Psi$

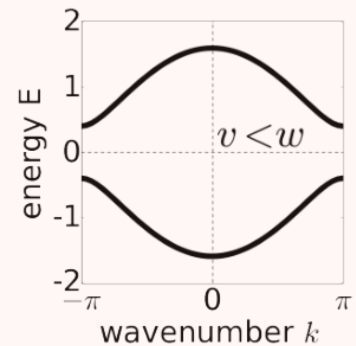
Wavefunctions: $\Psi_{\pm} = \begin{pmatrix} A \\ B \end{pmatrix}$

Energies: $E_{\pm}$

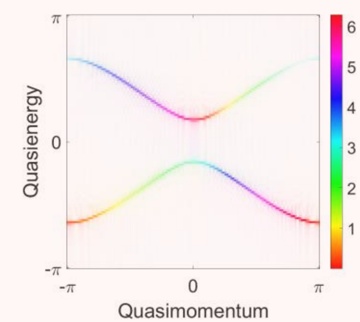
$\phi = \arg(A) - \arg(B)$



Topological: $v < w$

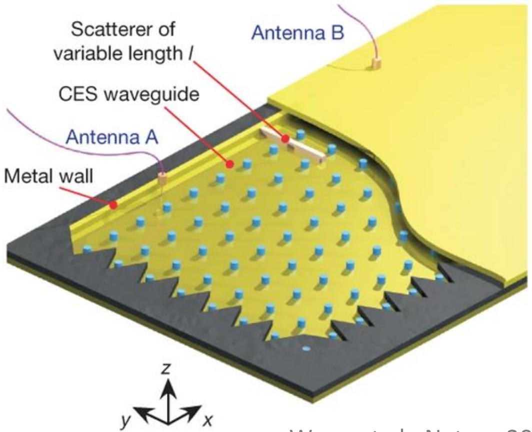


ϕ



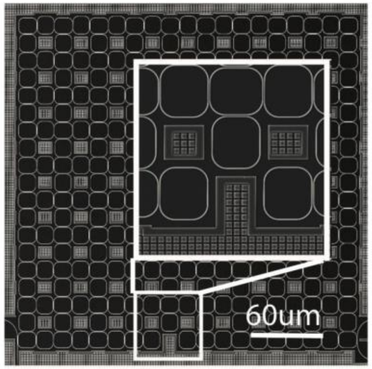
Topological photonics

Microwave resonators



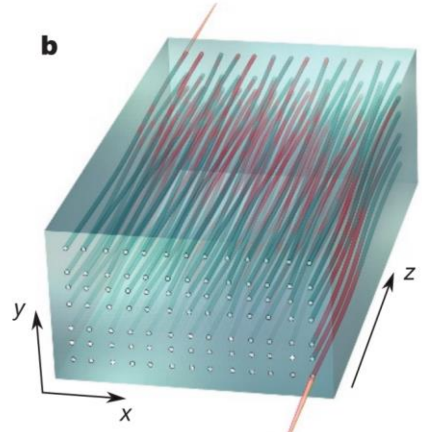
Wang et al., Nature 2009

Optical resonators



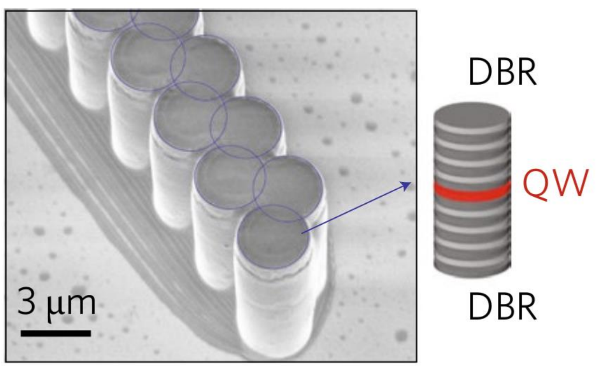
Hafezi et al., Nat. Phot. 2013

Waveguides



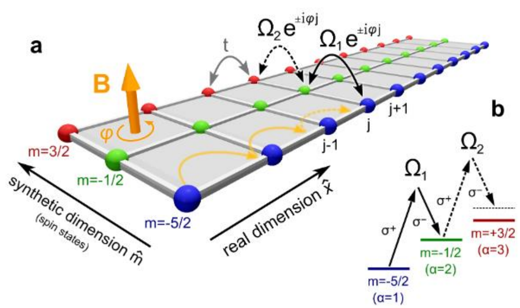
Zilberberg et al., Nature 2018

Exciton-polaritons



St-Jean et al., Nat. Phot. 2017

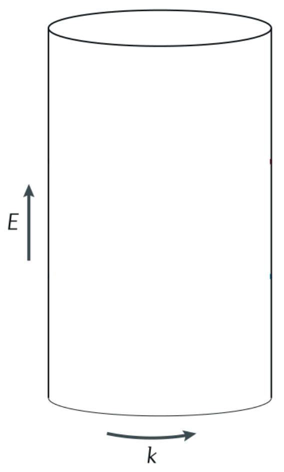
Cold atoms

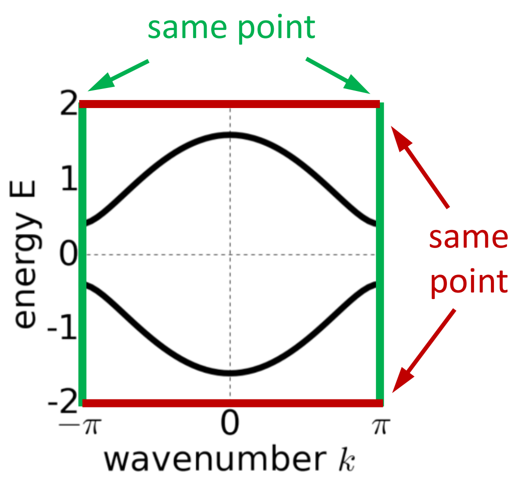


Mancini et al., Science 2015

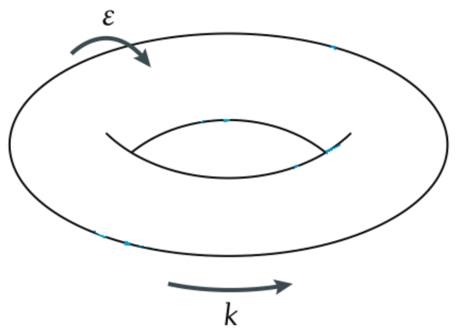
Floquet driving

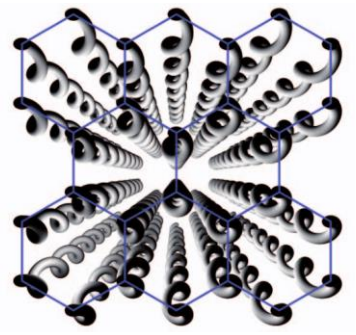
Crystal
periodicity in *space*



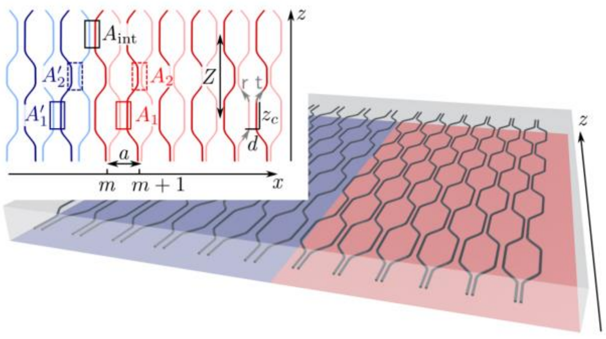


Floquet driving
periodicity in *time*

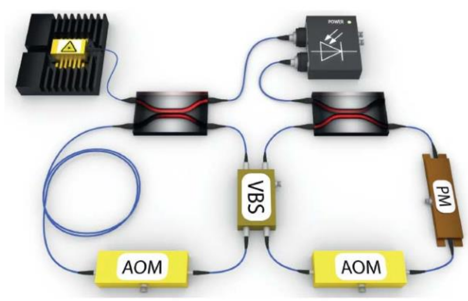




Rechtsmann et al., Nature 2013

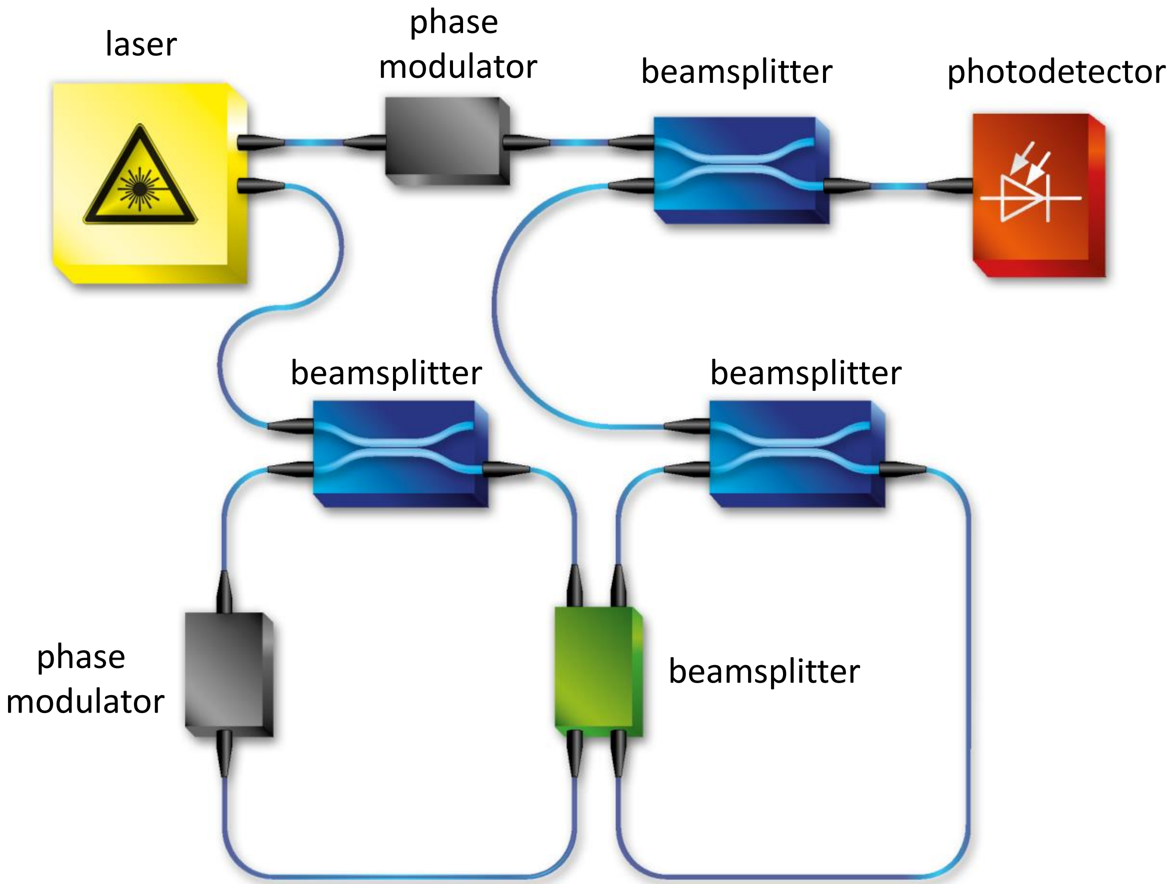


Bellec et al., Europhys. Lett. 2017

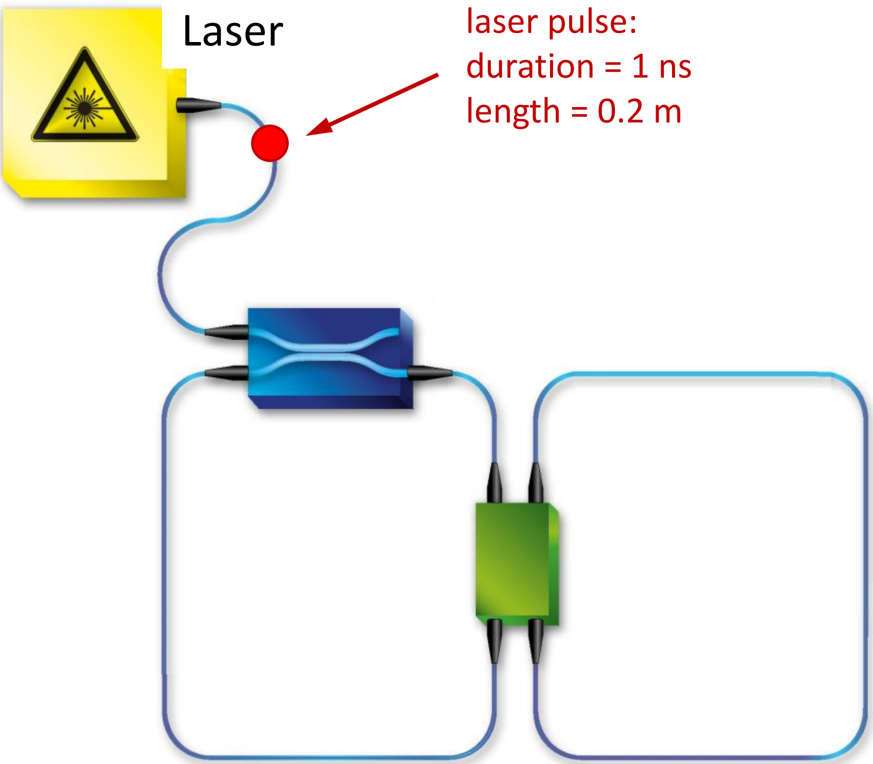


Weidemann et al., Science 2020
Regensburger et al., Nature 2012

Experimental platform

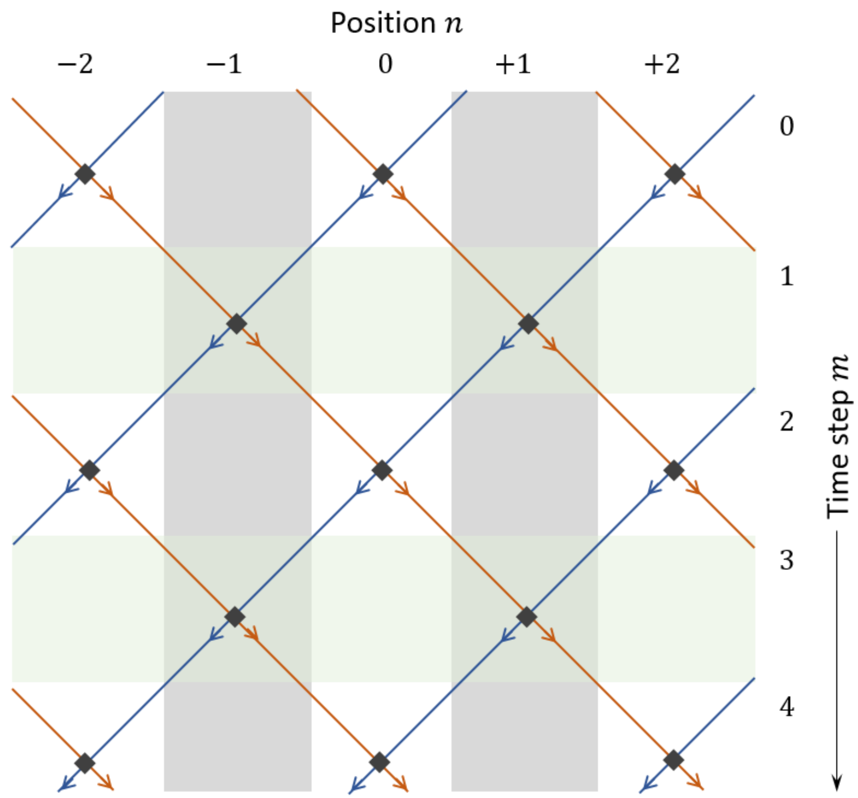


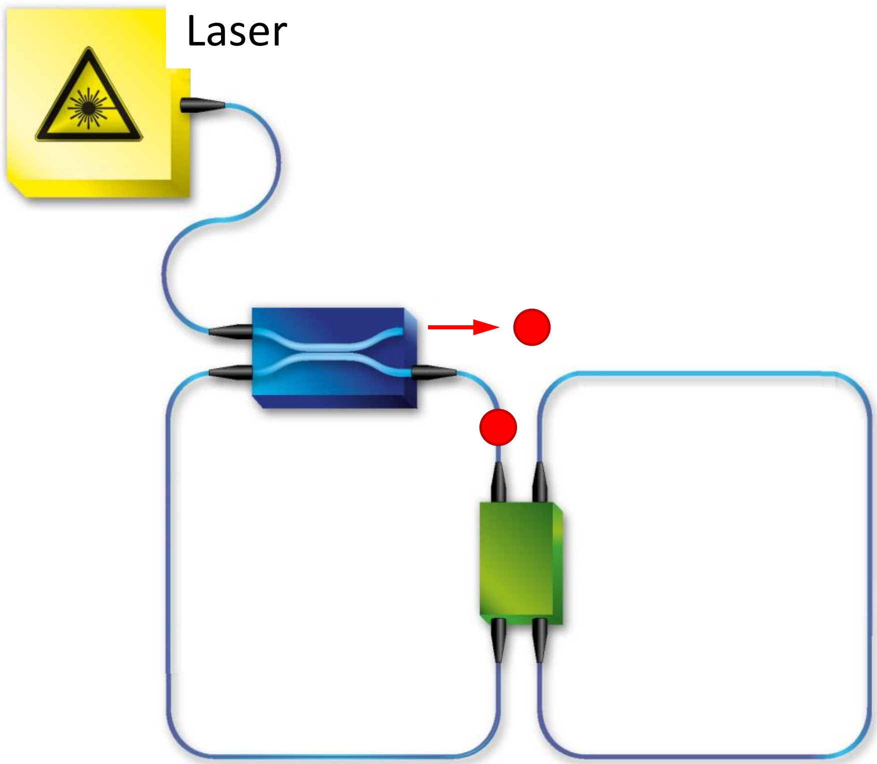
Mapping

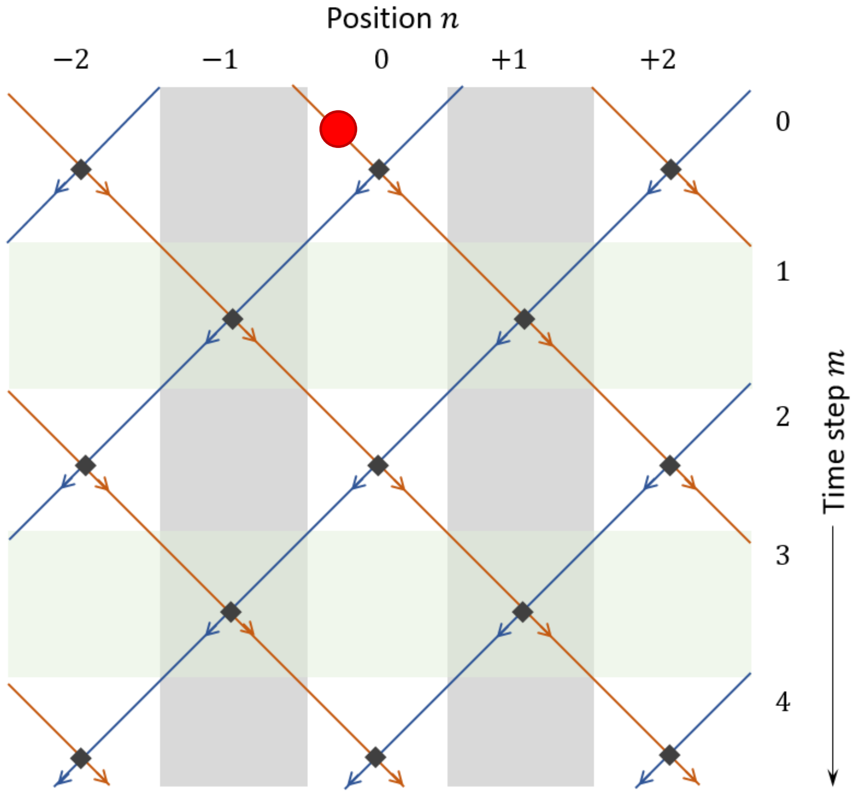


laser pulse:
duration = 1 ns
length = 0.2 m

rings ≈ 40 m each
length difference ≈ 0.5 m

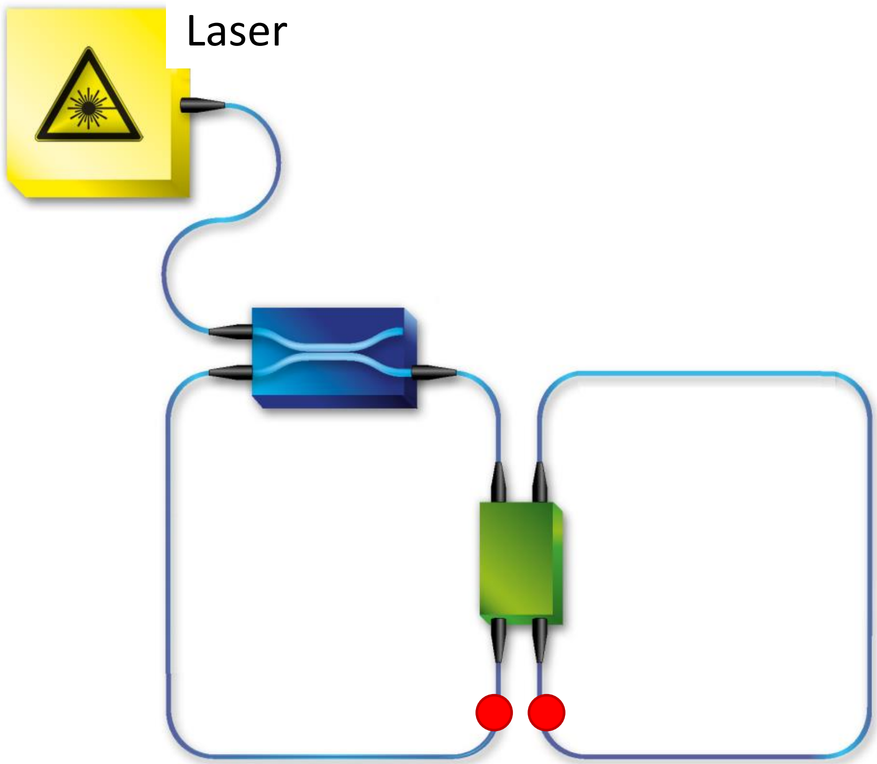


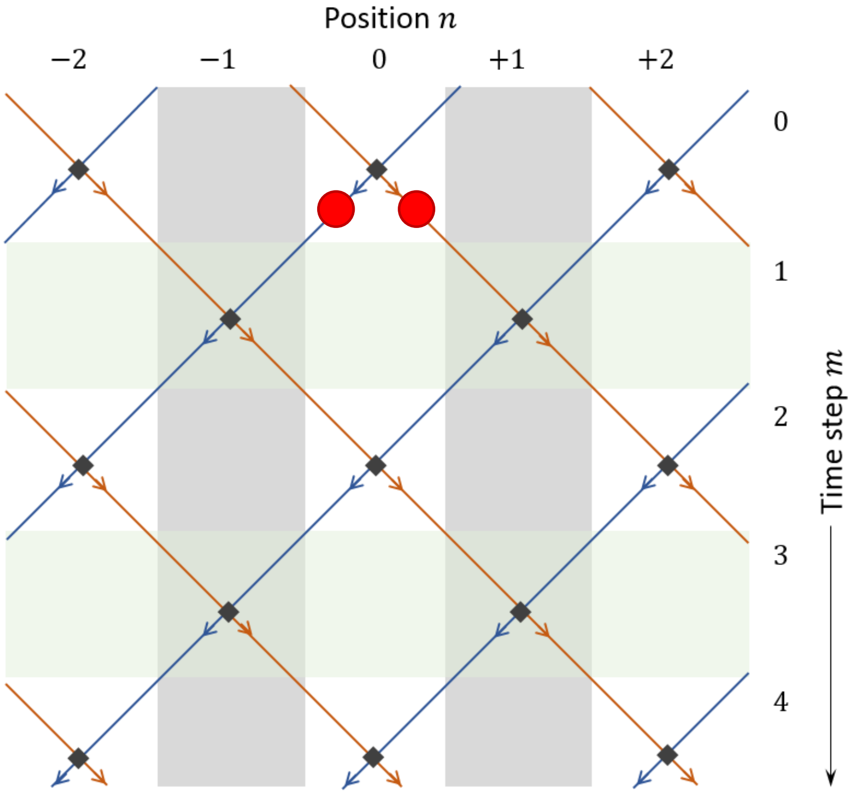

Laser


Position n
−2
−1
0
+1
+2
0
1
2
3
4
Time step m

Mapping

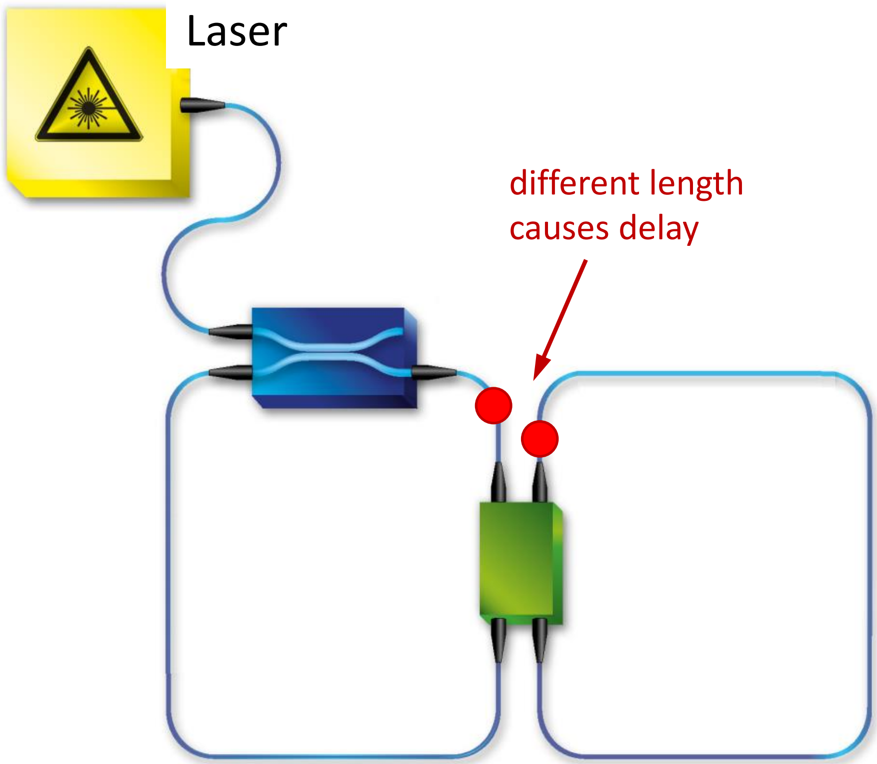
1st turn

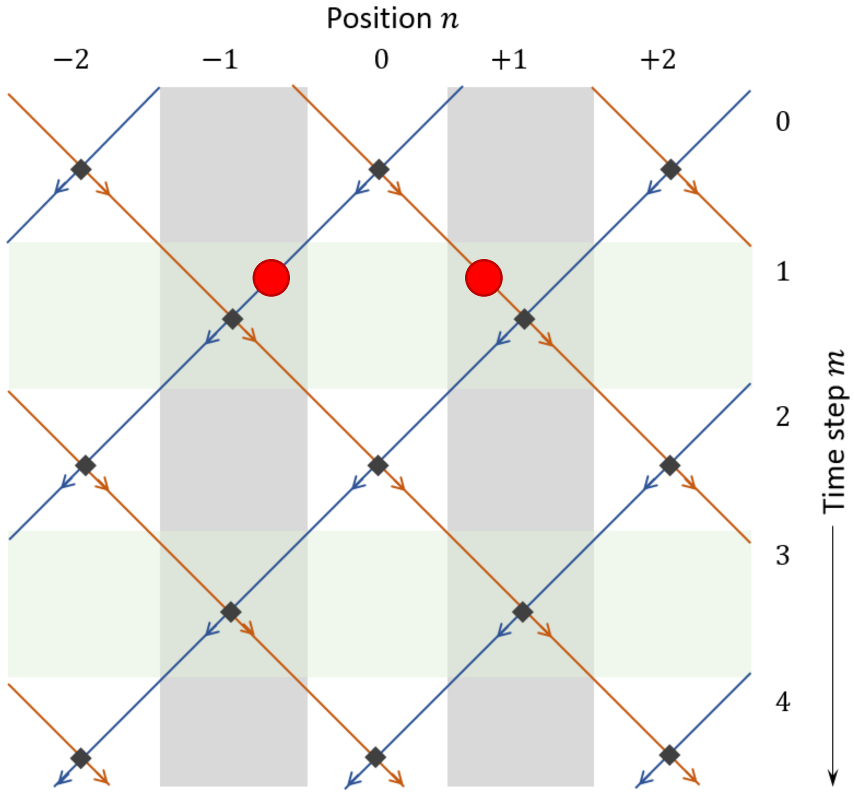




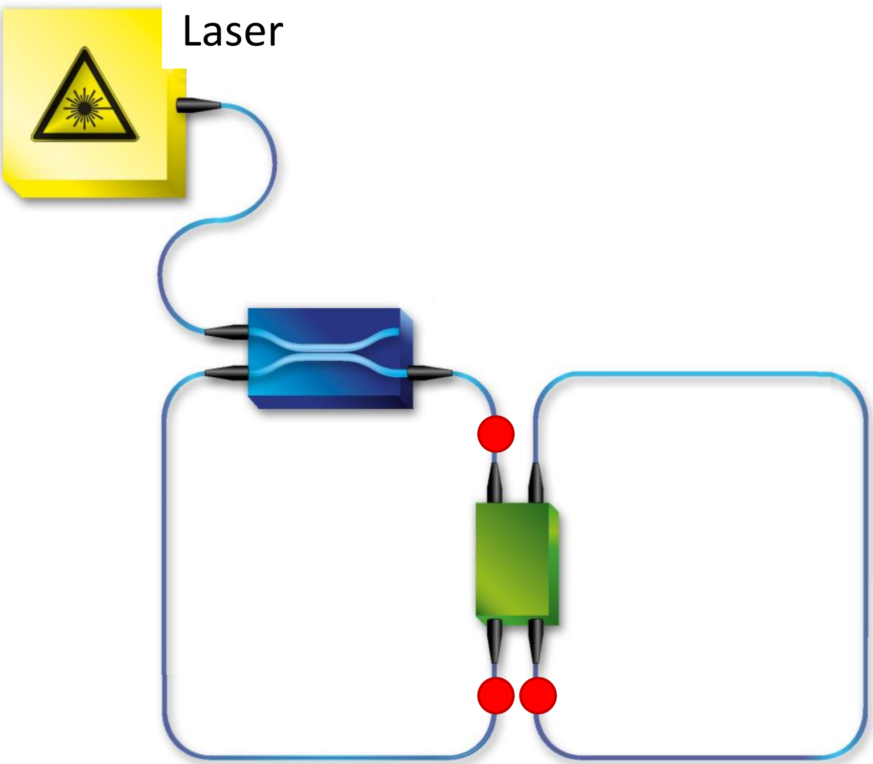
Mapping

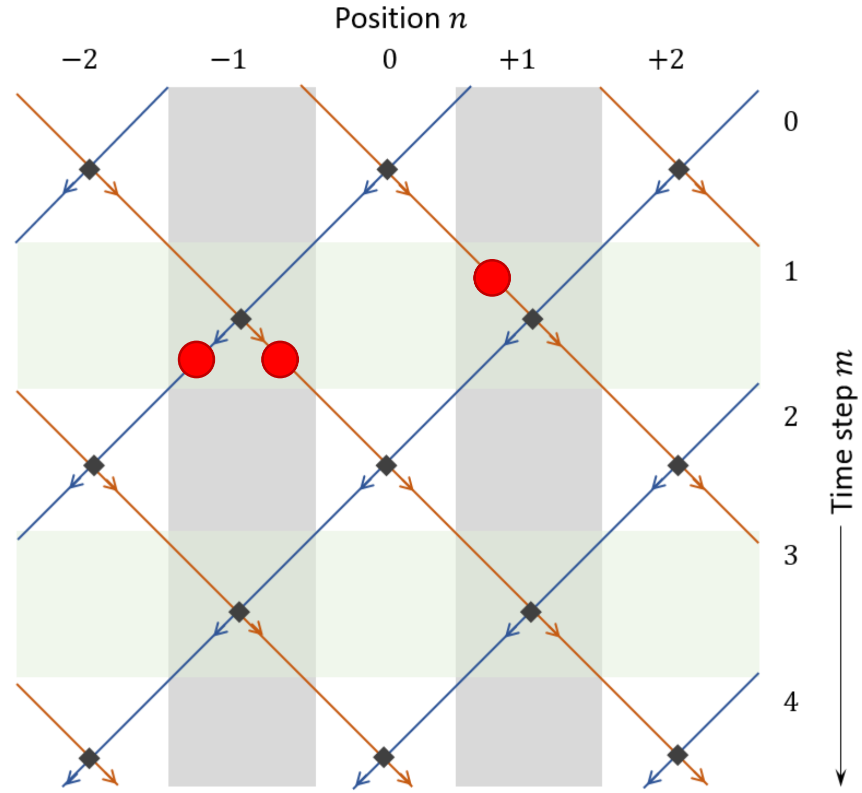
1st turn



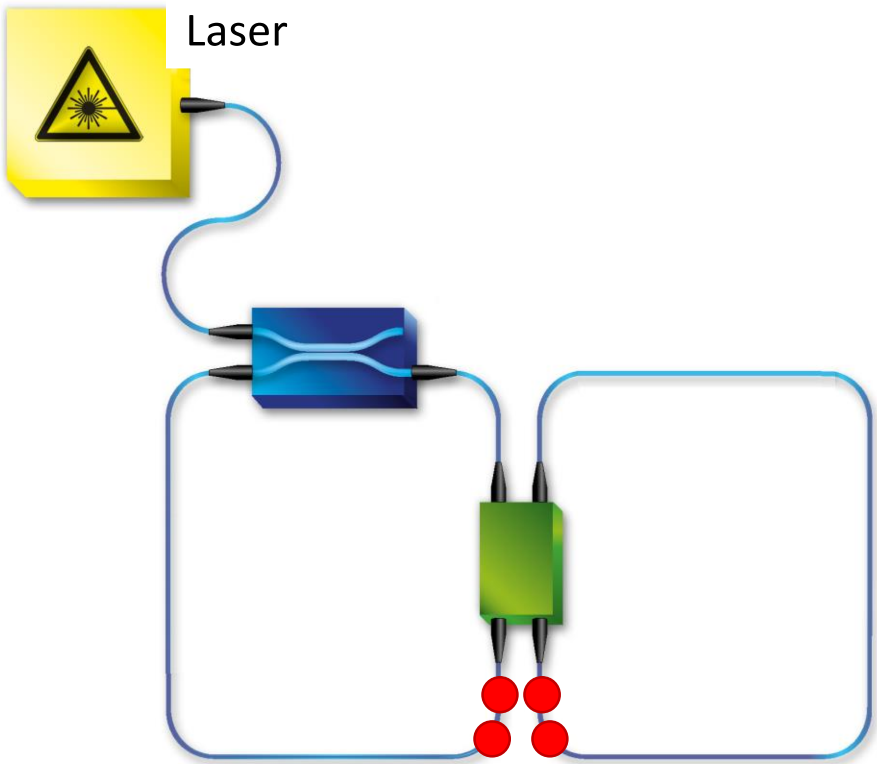


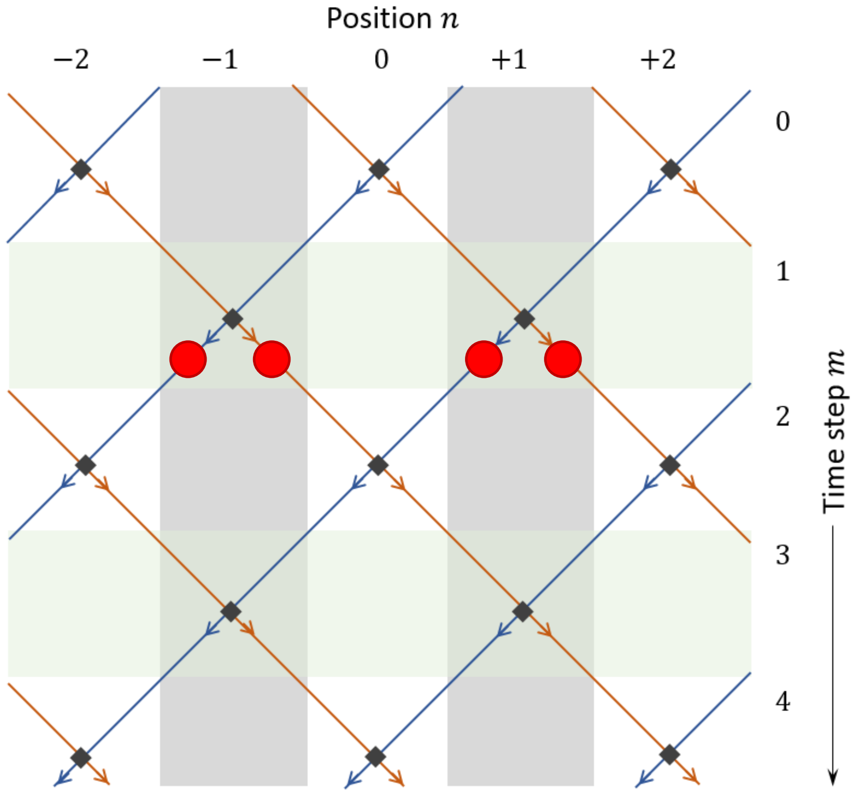
2^{nd} turn





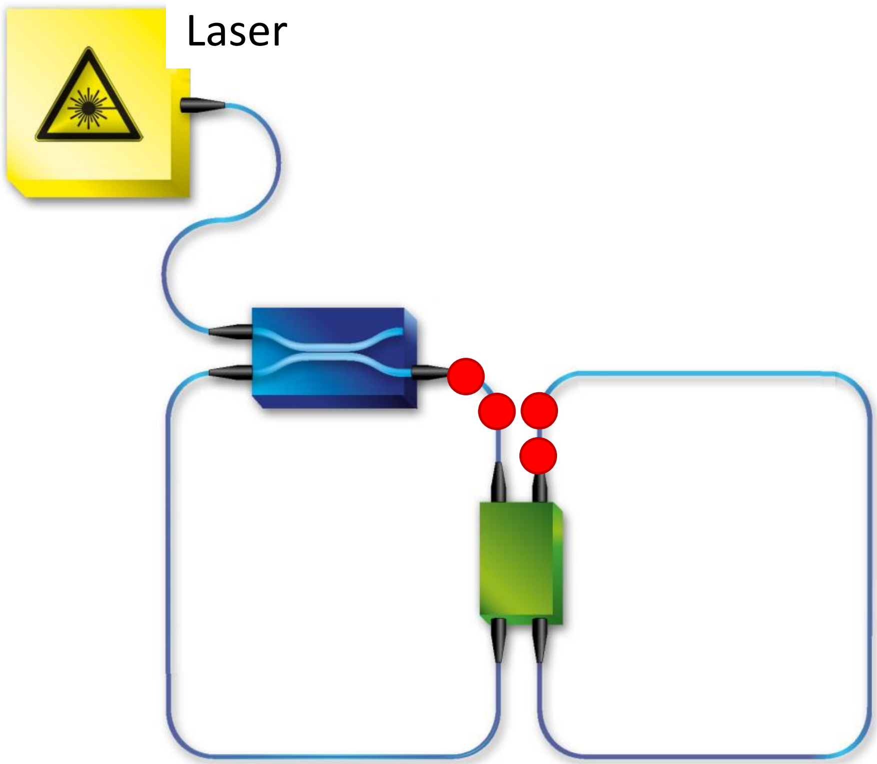
2nd turn

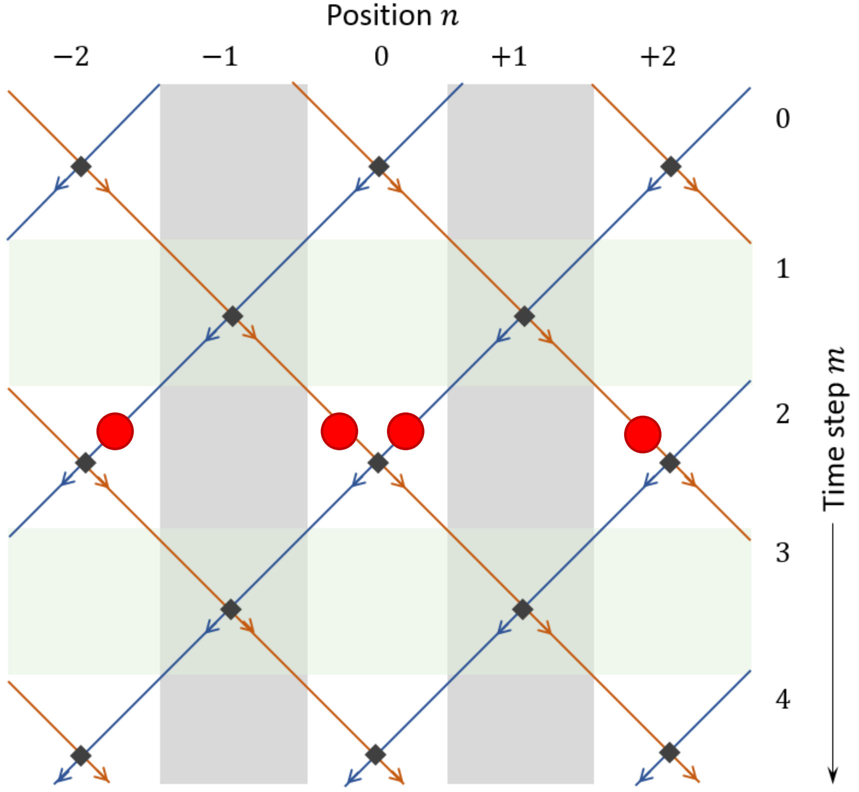




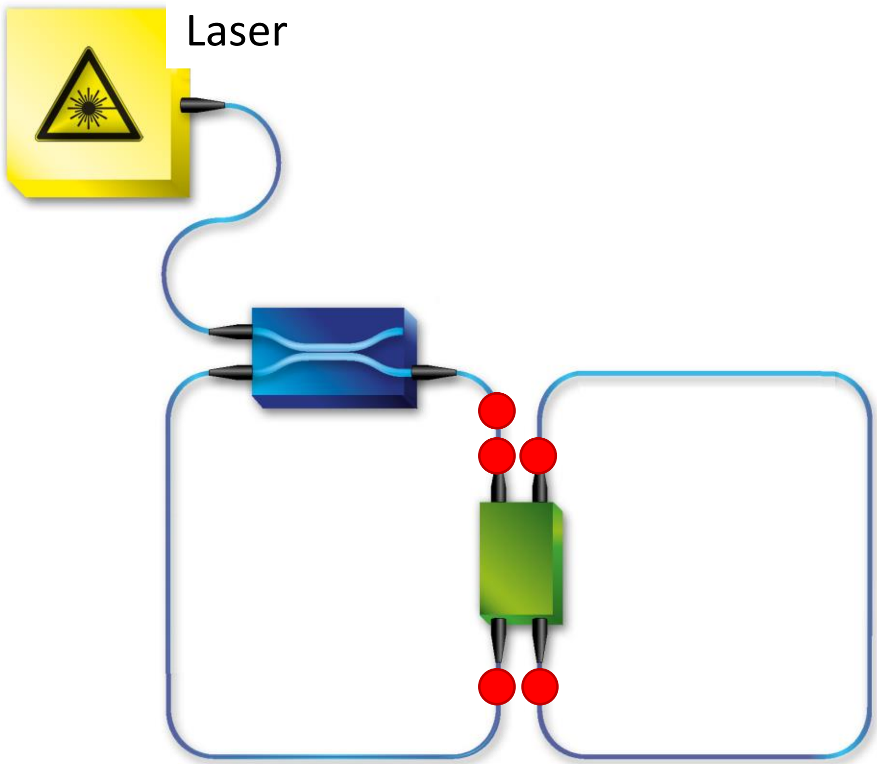
Mapping

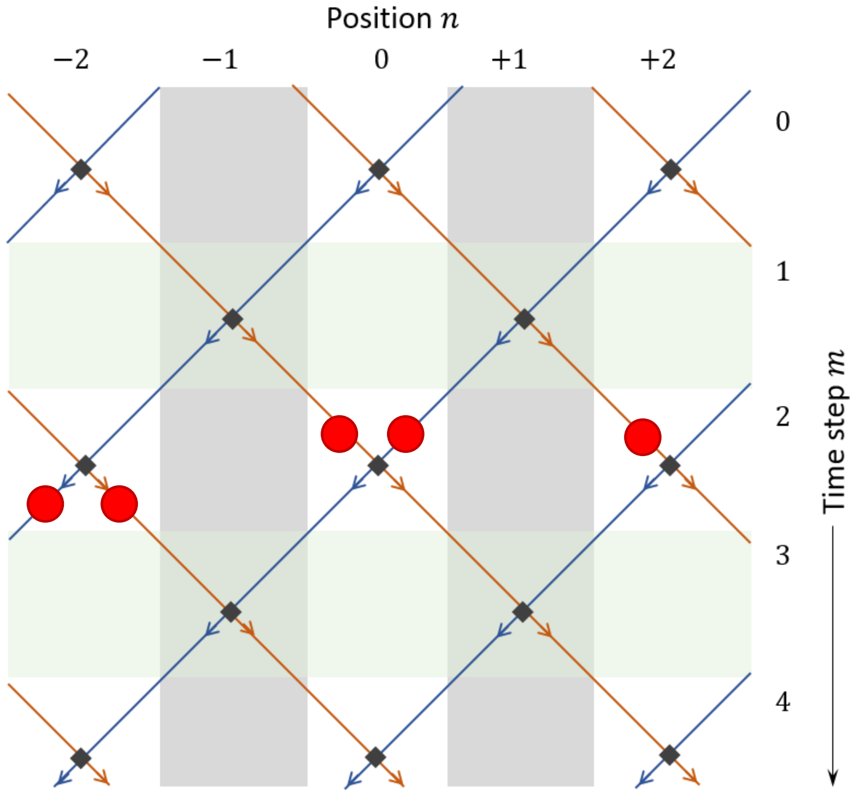
<u>2nd turn</u>





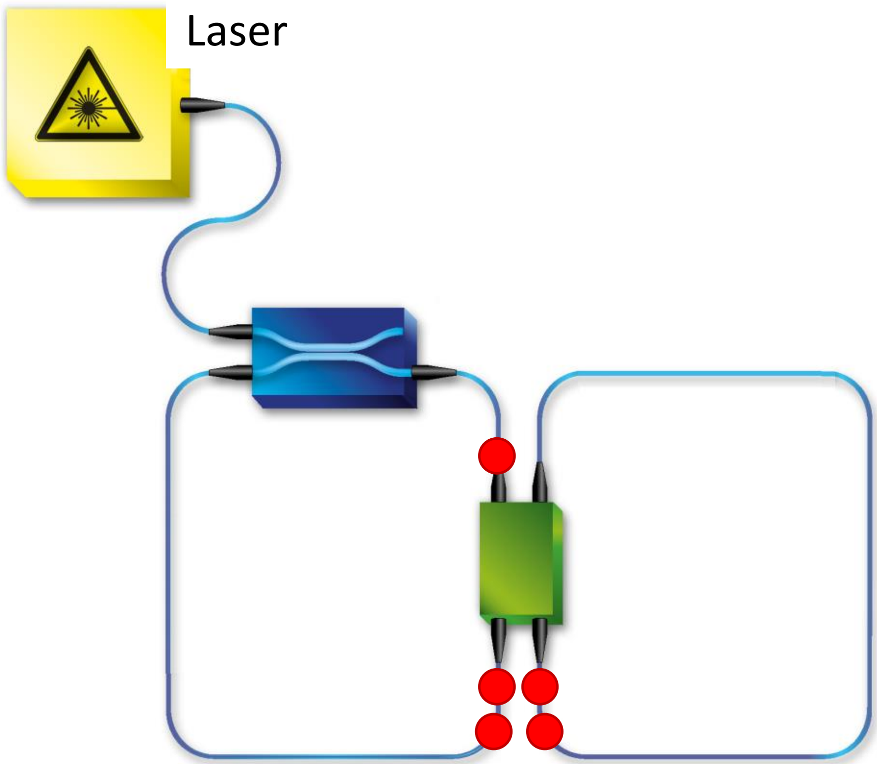
<u>3rd turn</u>

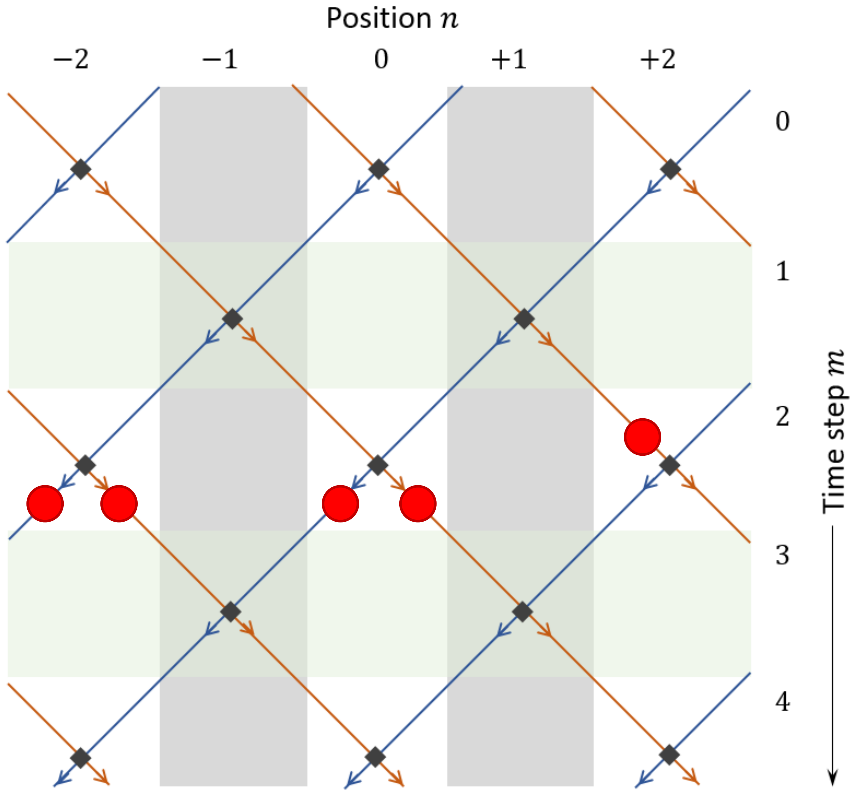




Mapping

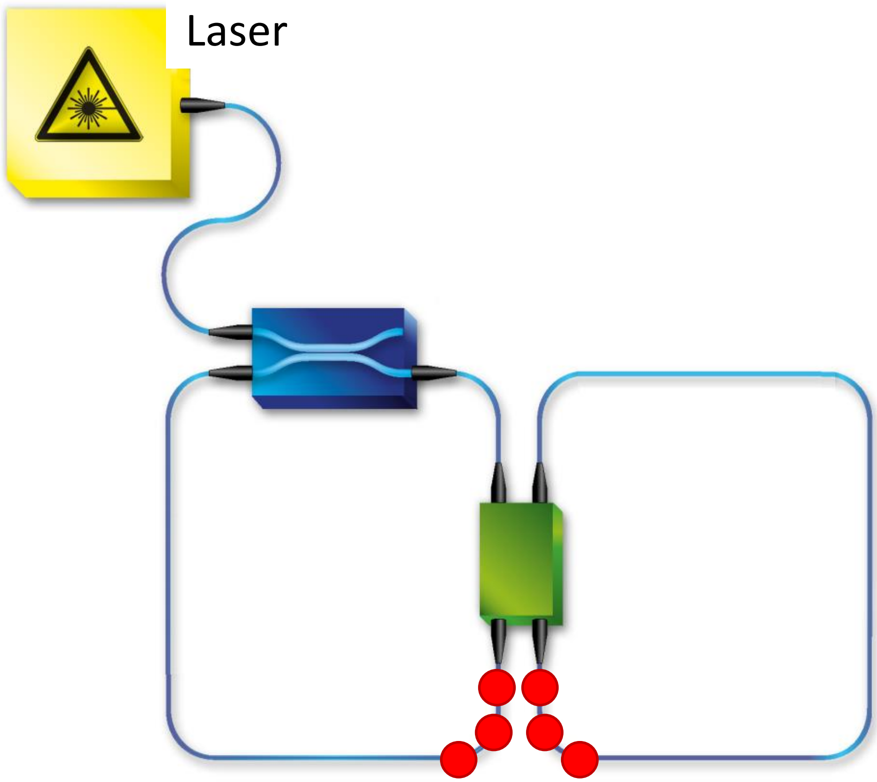
3rd turn

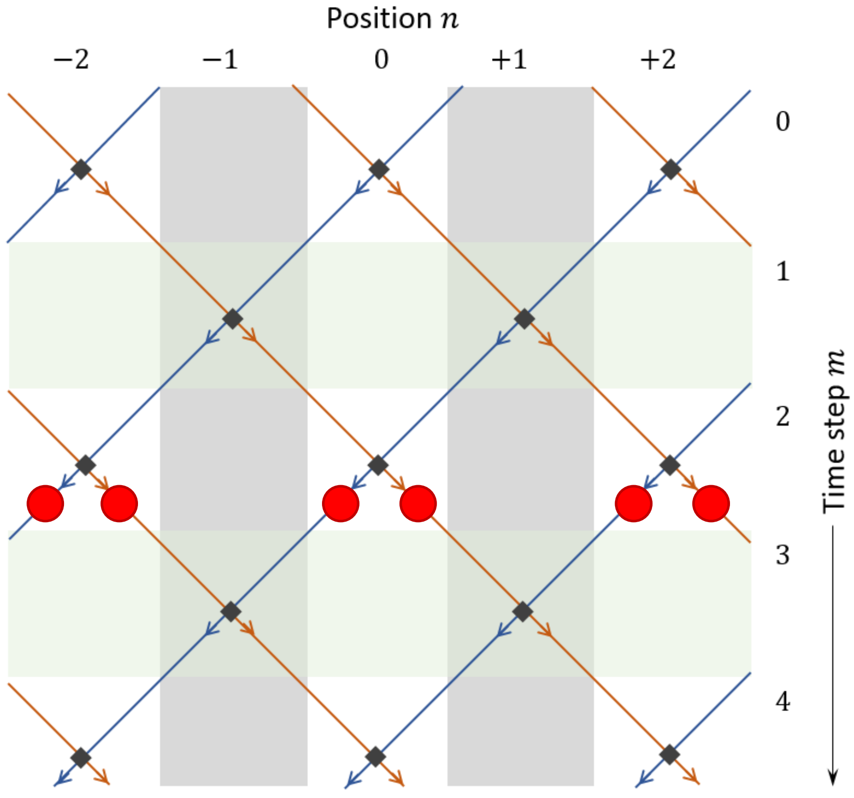




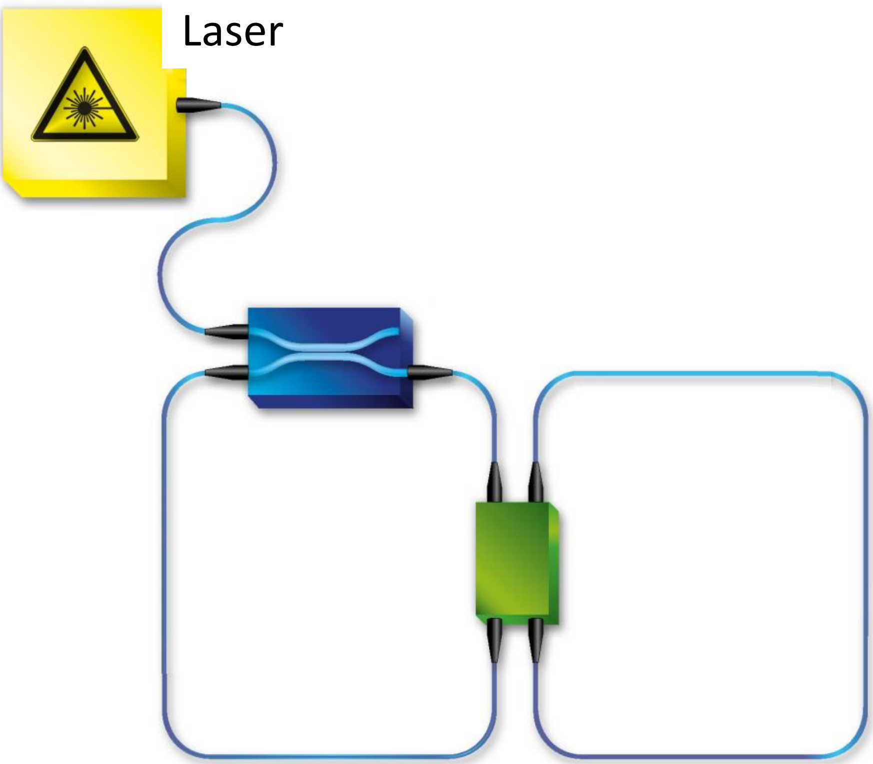
Mapping

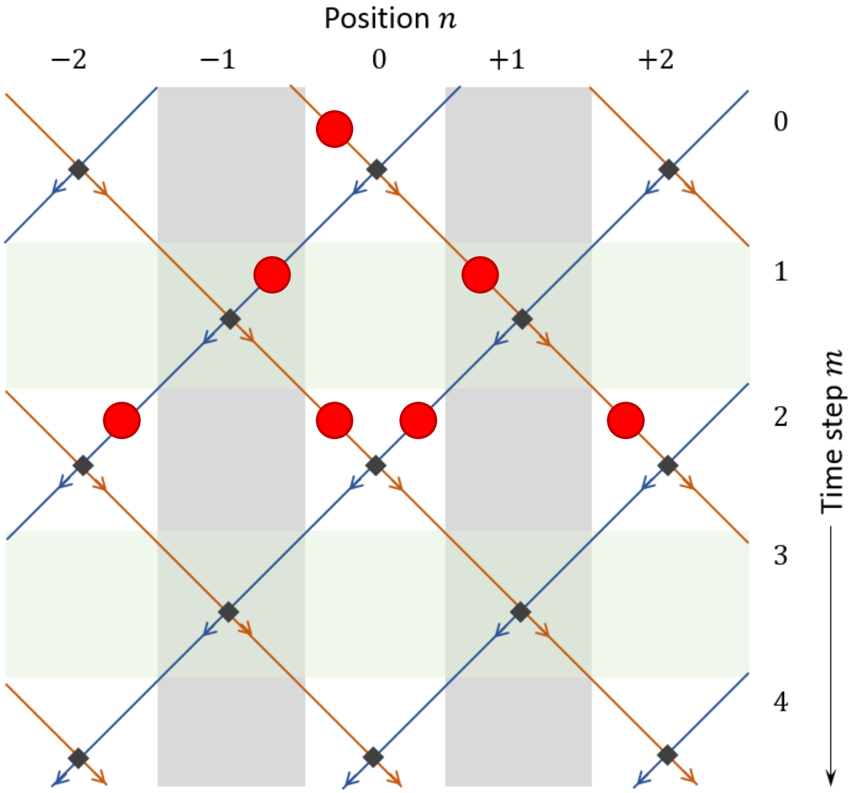
3rd turn





Mapping





Evolution equations

$$\alpha_n^{m+1} = \frac{1}{\sqrt{2}}\alpha_{n-1}^m + \frac{1}{\sqrt{2}} i\beta_{n-1}^m$$

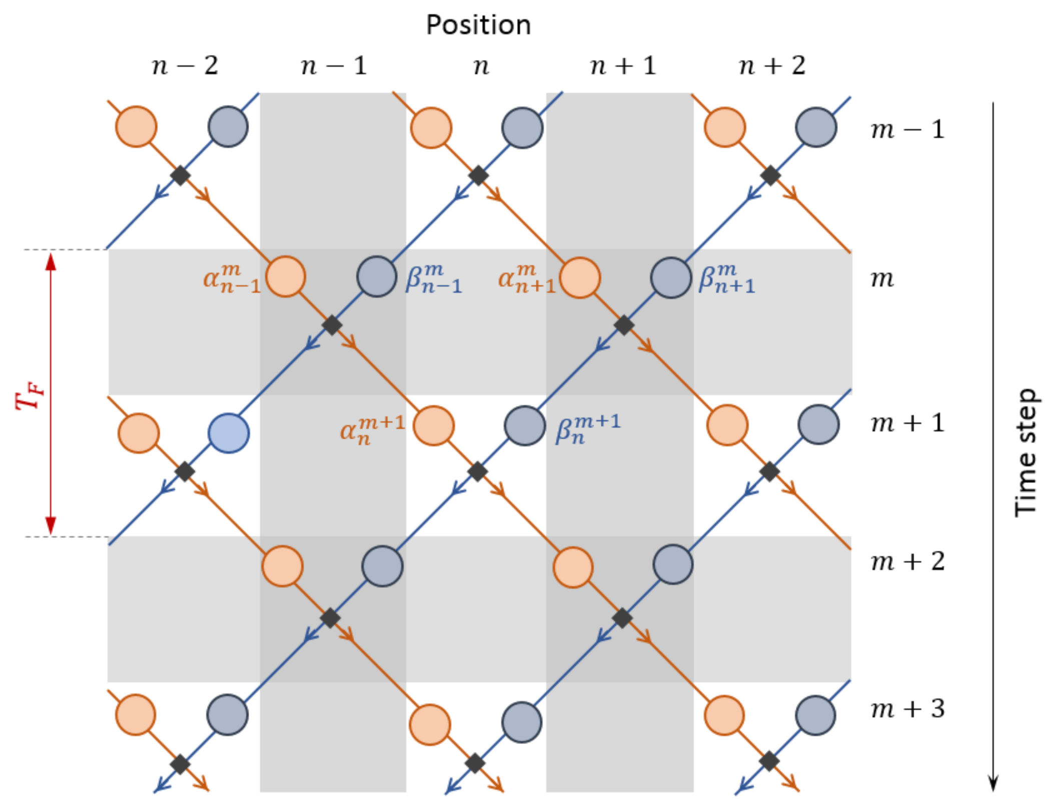
$$\beta_n^{m+1} = \frac{1}{\sqrt{2}} i\alpha_{n+1}^m + \frac{1}{\sqrt{2}}\beta_{n+1}^m$$

Floquet-Bloch ansatz:

$$\begin{pmatrix}\alpha_n^m \\ \beta_n^m\end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{i\frac{kn}{2}} e^{i\frac{Em}{2}}$$

Bands:

$$E = \pm\frac{1}{2}\cos^{-1}(1 - \cos k)$$



Evolution equations

$$\alpha_n^{m+1} = \frac{1}{\sqrt{2}}\alpha_{n-1}^m + \frac{1}{\sqrt{2}} i\beta_{n-1}^m$$

$$\beta_n^{m+1} = \frac{1}{\sqrt{2}} i\alpha_{n+1}^m + \frac{1}{\sqrt{2}}\beta_{n+1}^m$$

Floquet-Bloch ansatz:

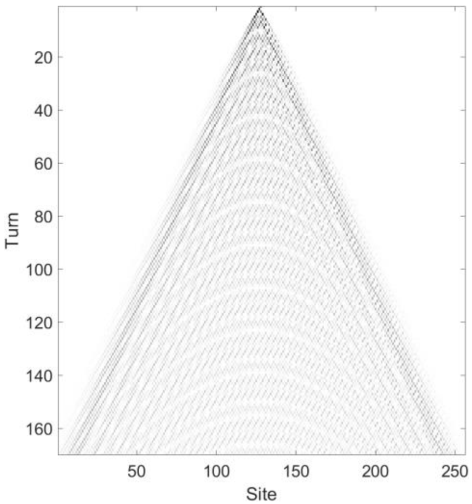
$$\begin{pmatrix}\alpha_n^m \\ \beta_n^m\end{pmatrix} = \begin{pmatrix}A \\ B\end{pmatrix} e^{i\frac{kn}{2}} e^{i\frac{Em}{2}}$$

Bands:

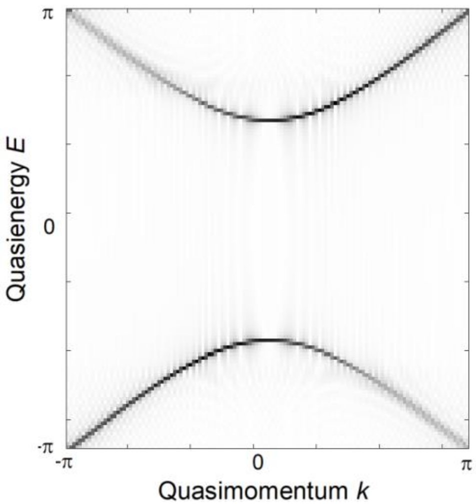
$$E = \pm\frac{1}{2}\cos^{-1}(1-\cos k)$$

Simulations

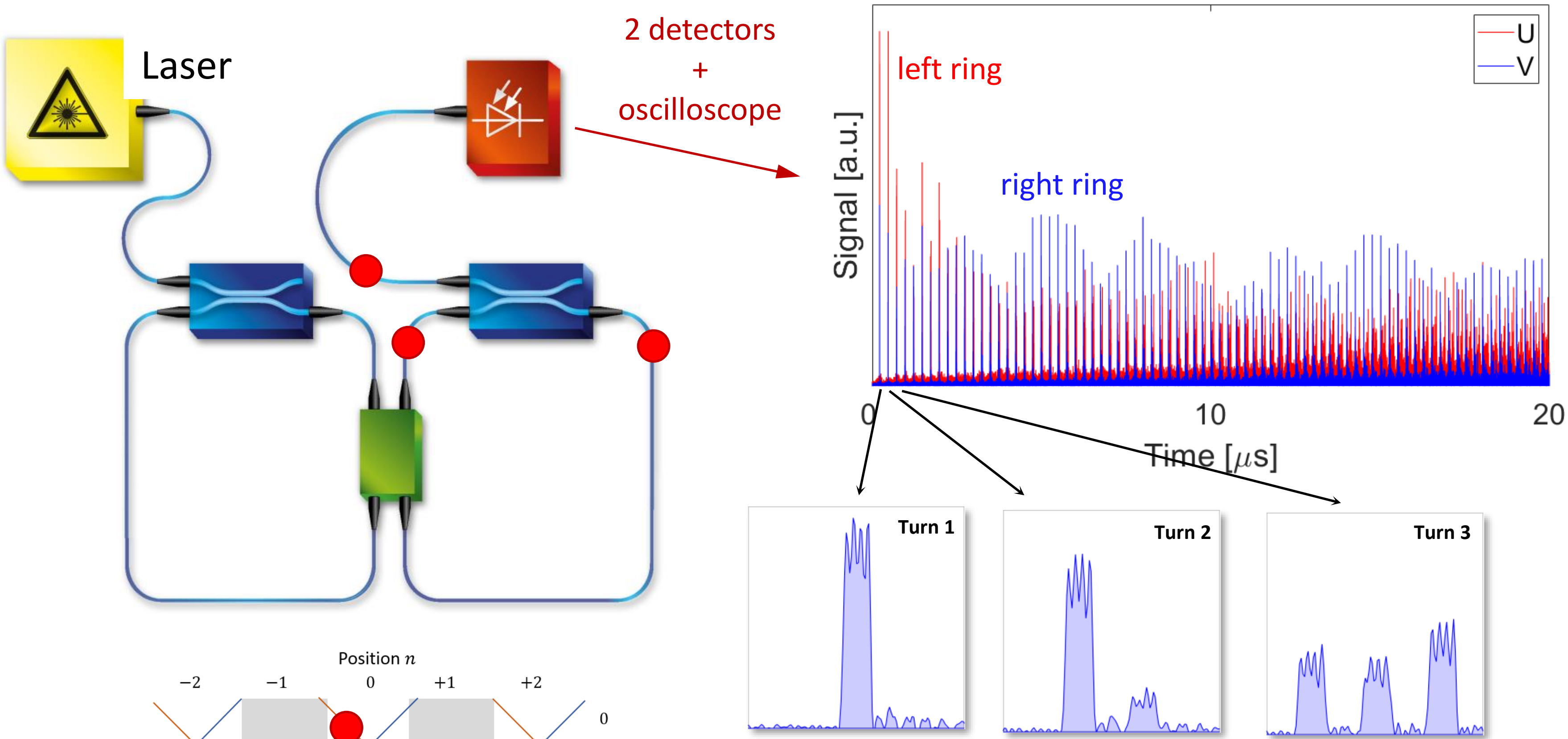
Light walk

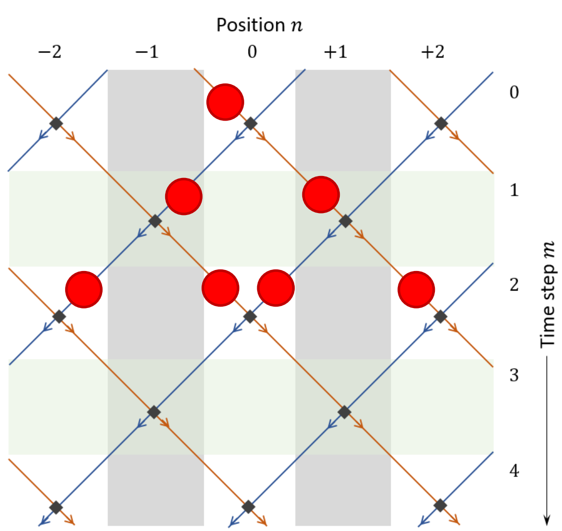


Band structure
(Fourier transform of the walk)

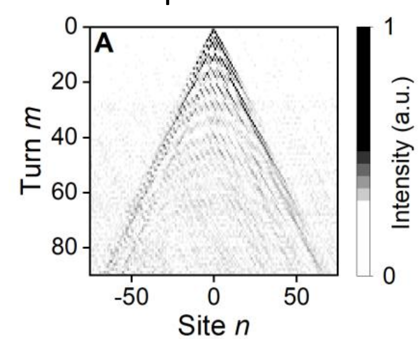


Measuring the intensity

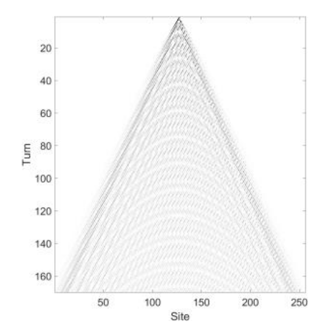




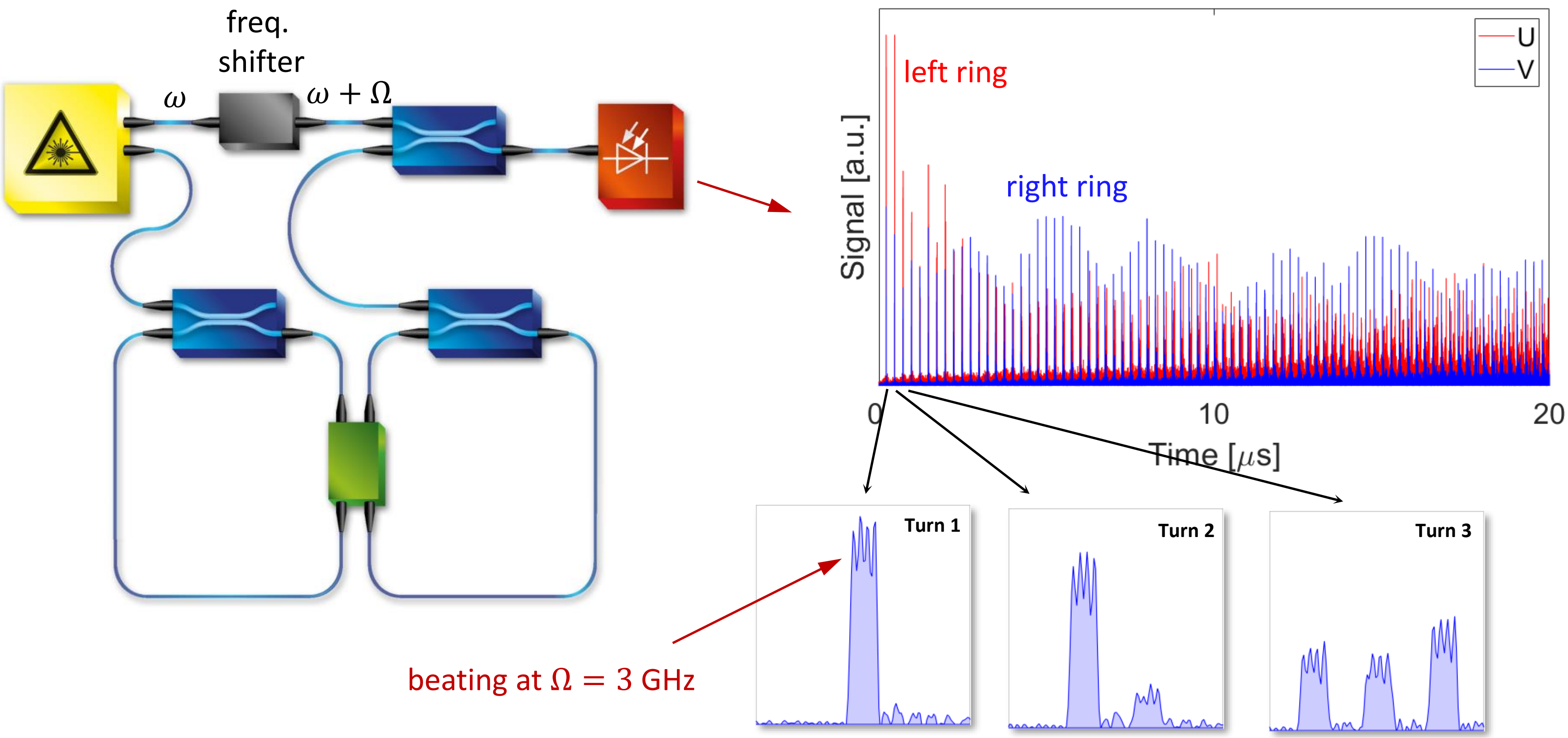
Experiment:



Simulations:



Measuring the phase

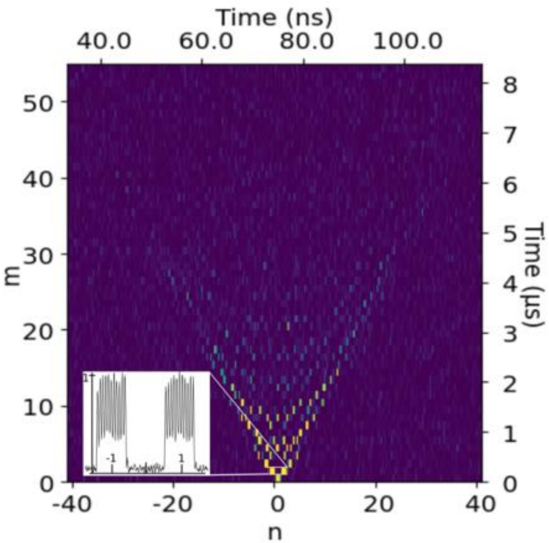


Beating:

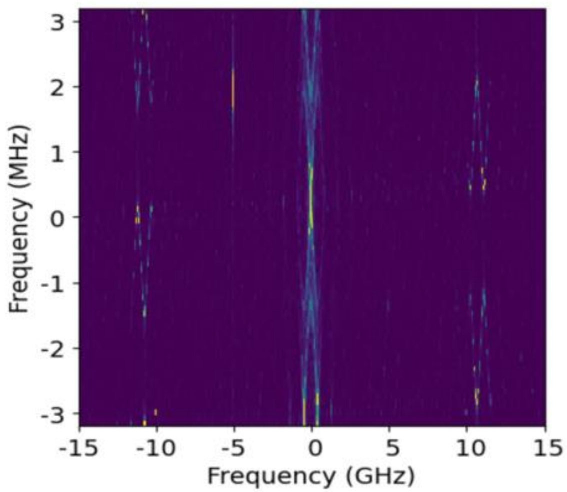
$$I = \left| A_s e^{i\varphi_s} e^{i\omega t} + A_{LO} e^{i(\omega+\Omega)t} \right|^2 = A_s^2 + A_{LO}^2 + 2 A_s A_{LO} \cos(\Omega t + \varphi_s)$$

Measuring the phase

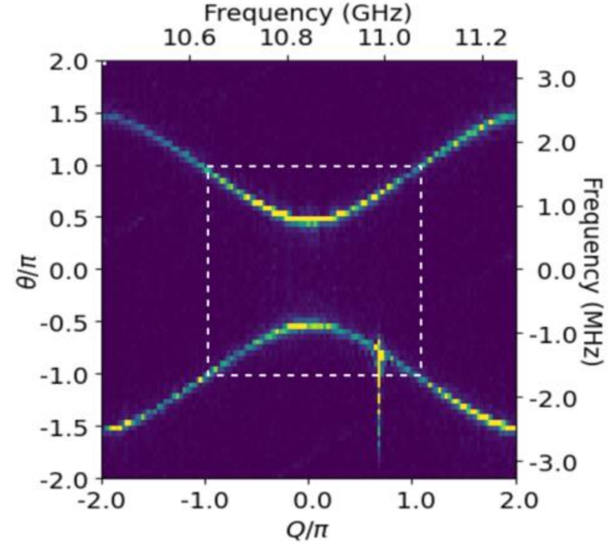
Real space



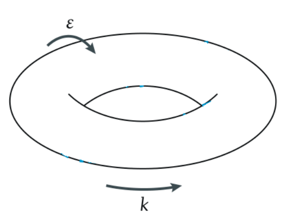
Reciprocal space (Fourier transform)



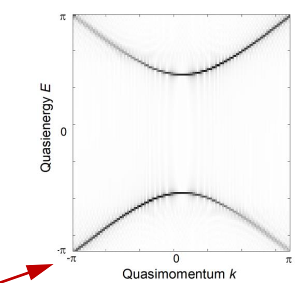
Zoom in at LO frequency



Lechevalier et al., Commun. Phys. 2021

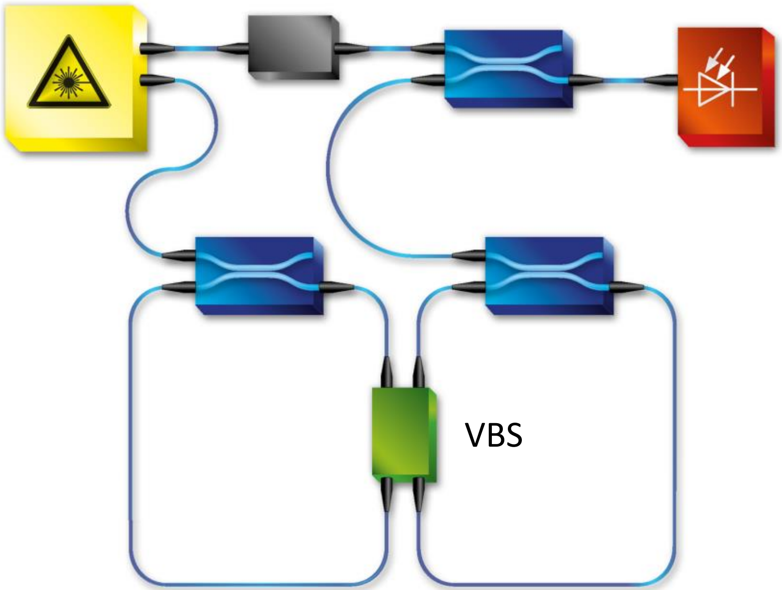


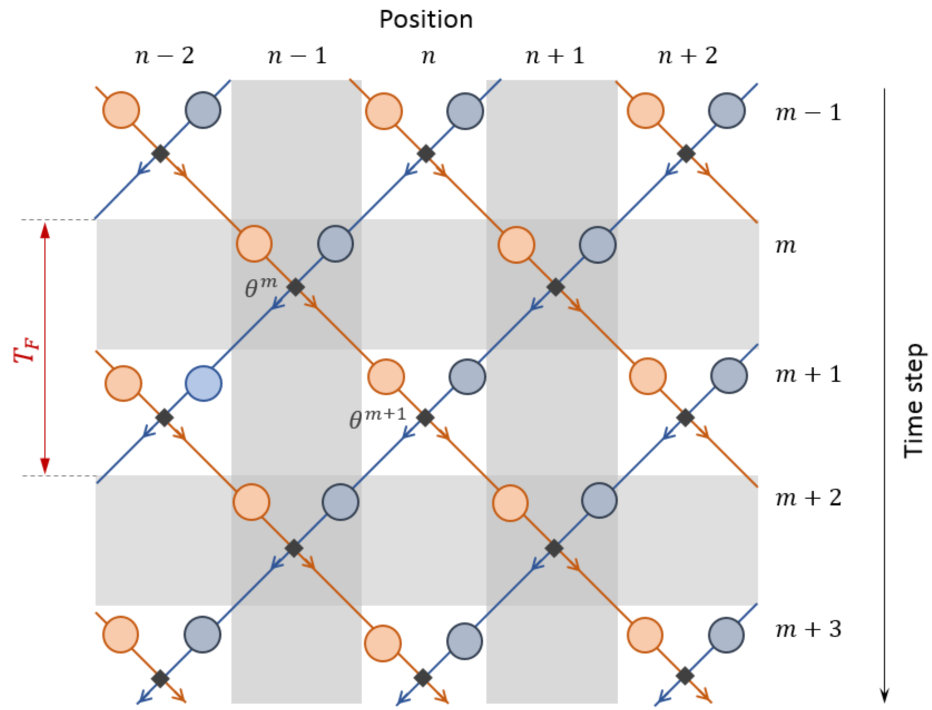
Simulations:



2 touching bands

Splitting the bands: VBS





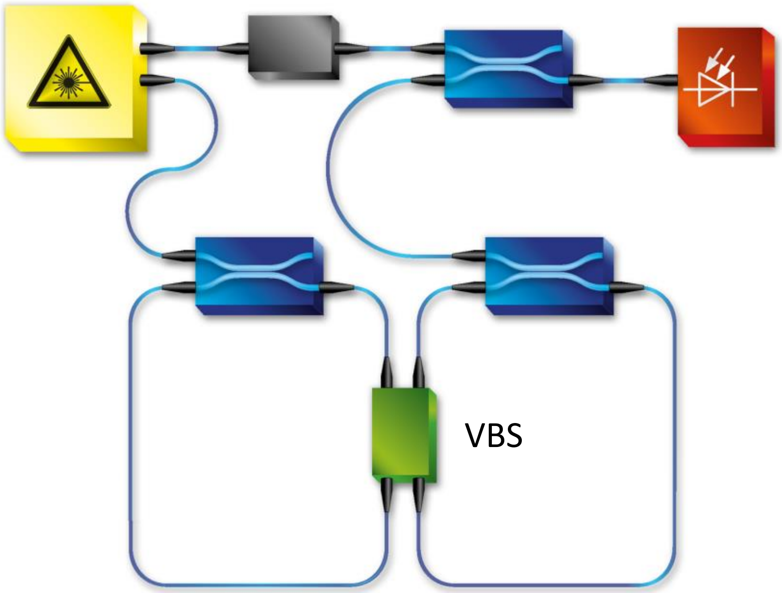
$$\alpha_n^{m+1} = \cos\theta_m\, \alpha_{n-1}^m + i\sin\theta_m\, \beta_{n-1}^m$$

$$\beta_n^{m+1} = i\sin\theta_m\, \alpha_{n+1}^m + \cos\theta_m\, \beta_{n+1}^m$$

e.g. $\theta_m = \pi/4$ means $\sin\theta_m = \cos\theta_m = \frac{1}{\sqrt{2}}$
as for 50:50 beamsplitter

We use 2-step driving: θ_1 on odd steps, θ_2 on even steps

Splitting the bands: VBS



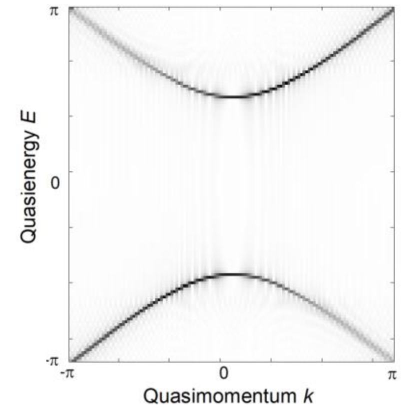
$$\alpha_n^{m+1} = \cos\theta_m\,\alpha_{n-1}^m + i\sin\theta_m\,\beta_{n-1}^m$$

$$\beta_n^{m+1} = i\sin\theta_m\,\alpha_{n+1}^m + \cos\theta_m\,\beta_{n+1}^m$$

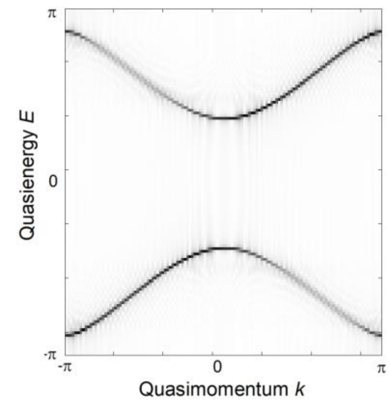
e.g. $\theta_m = \pi/4$ means $\sin\theta_m = \cos\theta_m = \frac{1}{\sqrt{2}}$
as for 50:50 beamsplitter

We use 2-step driving: θ_1 on odd steps, θ_2 on even steps

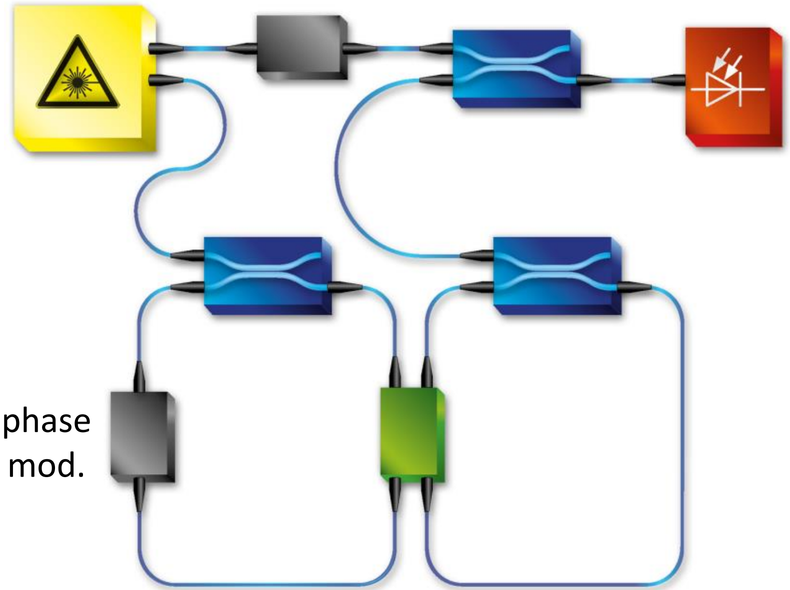
$\theta_1 = \theta_2 = \pi/4$ touching bands

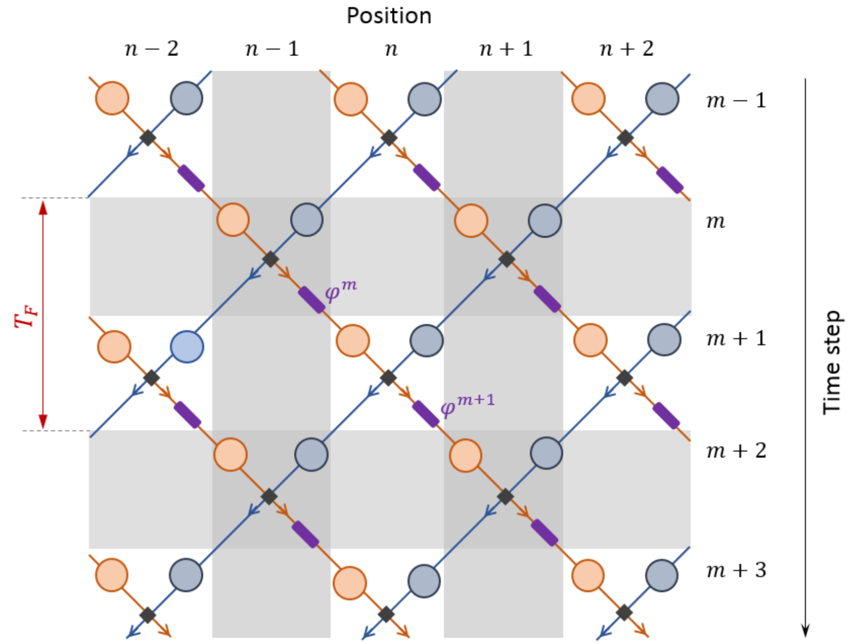


$\theta_1 = \pi/4$
$\theta_2 = \pi/4 - 0.4$ gap opening



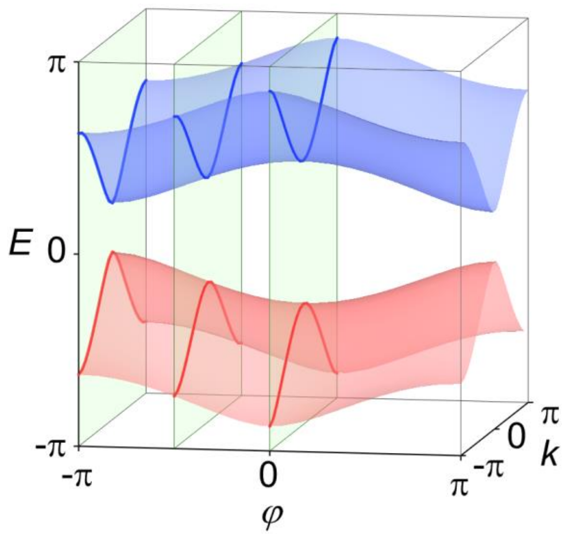
Expanding to 2D: phase





$$\alpha_n^{m+1} = (\cos\theta_m\, \alpha_{n-1}^m + i\sin\theta_m\, \beta_{n-1}^m)e^{i\varphi_m}$$

$$\beta_n^{m+1} = i\sin\theta_m\, \alpha_{n+1}^m + \cos\theta_m\, \beta_{n+1}^m$$



Again, 2 steps: $\varphi_1 = c_1\varphi$ and $\varphi_2 = c_2\varphi$

c_1 and c_2 are integers

$\varphi \in [-\pi, \pi]$ as a new dimension

Let's introduce integers

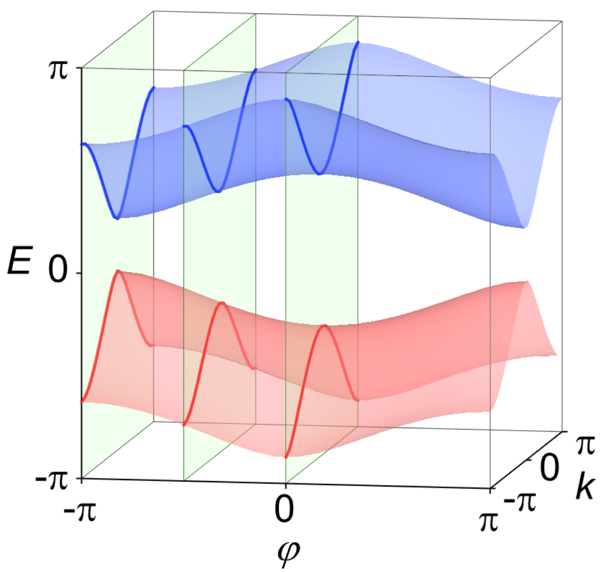
$$K = (c_1 + c_2)/2 \qquad \Delta = (c_1 - c_2)/2$$

so that

$$\varphi_1 = (K + \Delta)\varphi \qquad \varphi_2 = (K - \Delta)\varphi$$

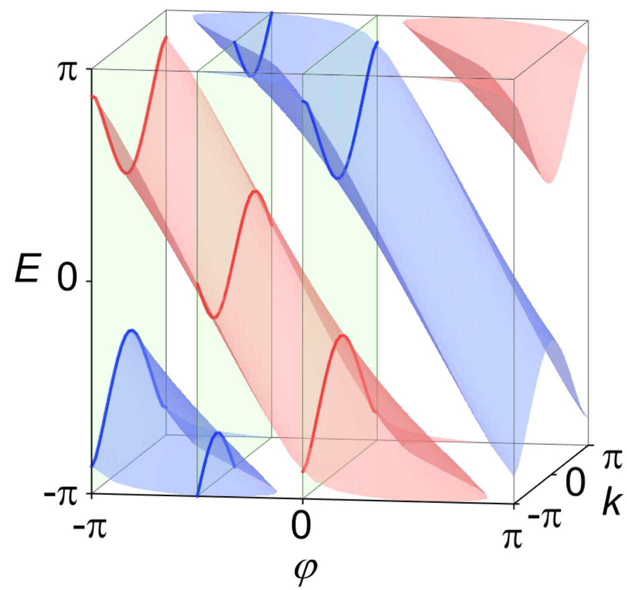
$K = 0$

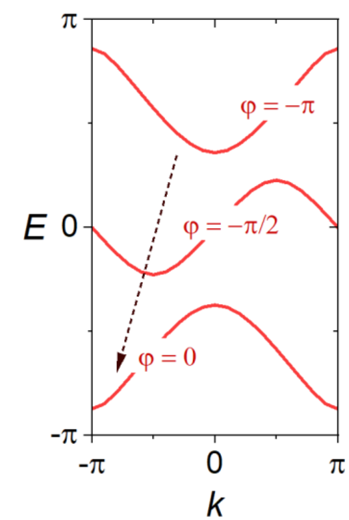
$\varphi_1 + \varphi_2 = 0$



$K = 1$

$\varphi_1 + \varphi_2 \neq 0$





Floquet winding metal

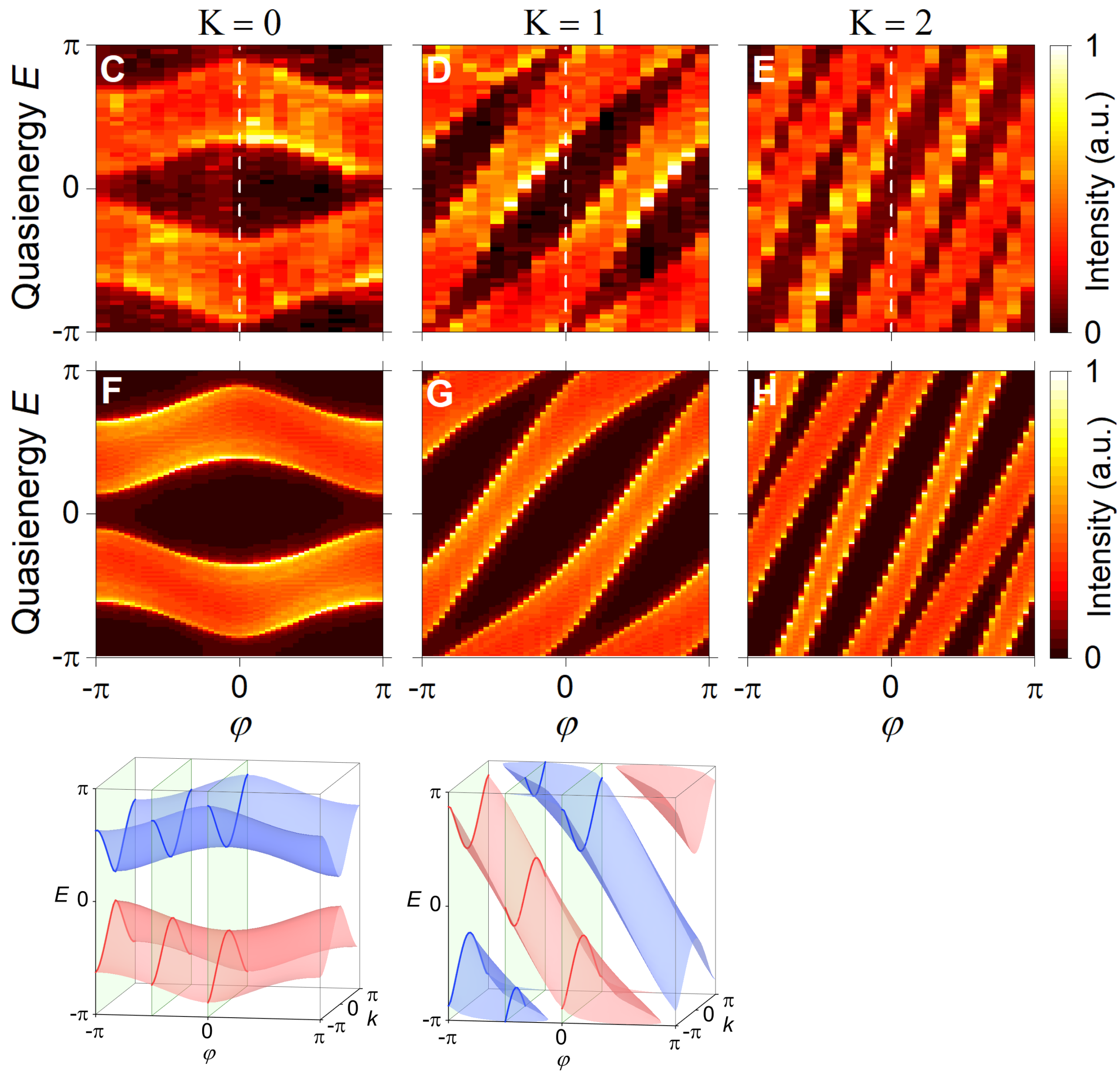
see Upreti et al., Phys. Rev. Lett. 2020

Band tomography

Experiment

Theory

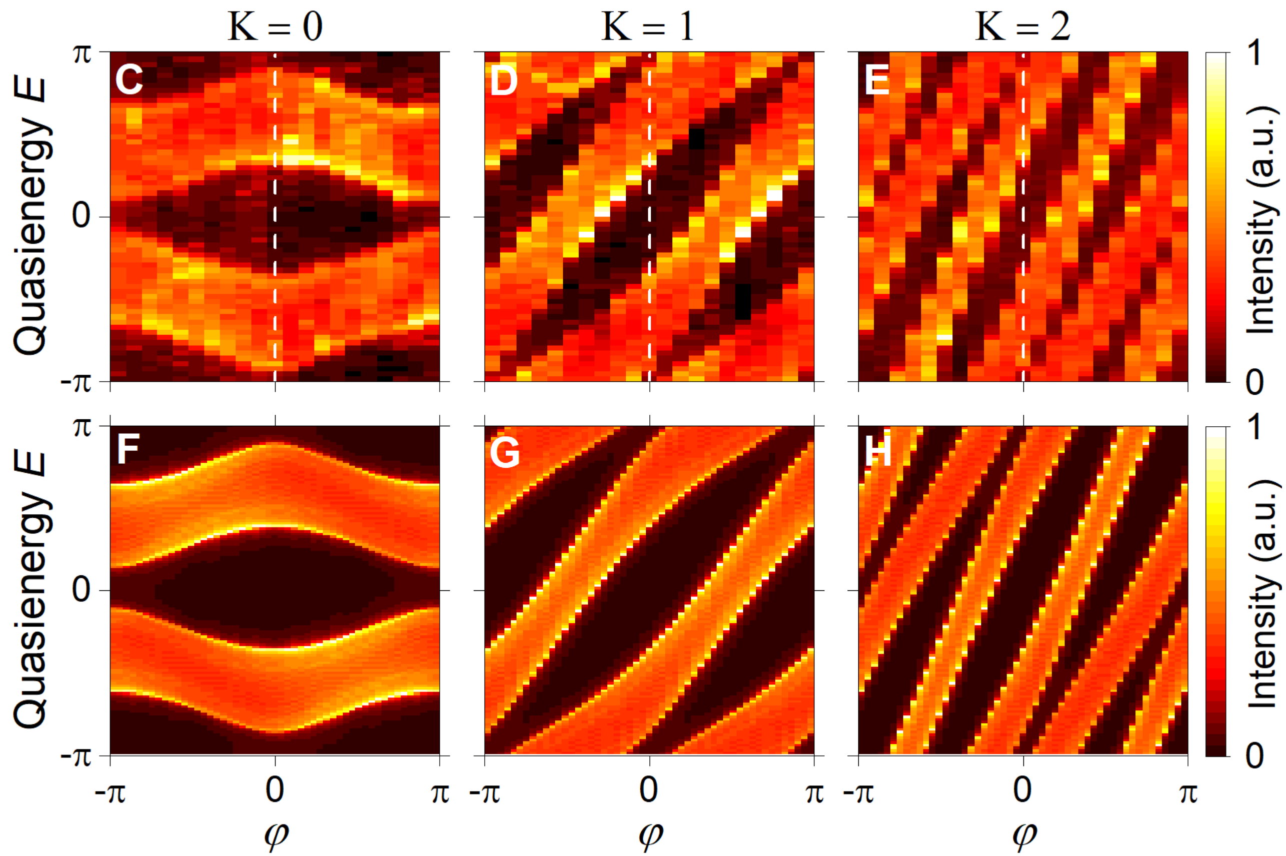
3D model



Band tomography

Experiment

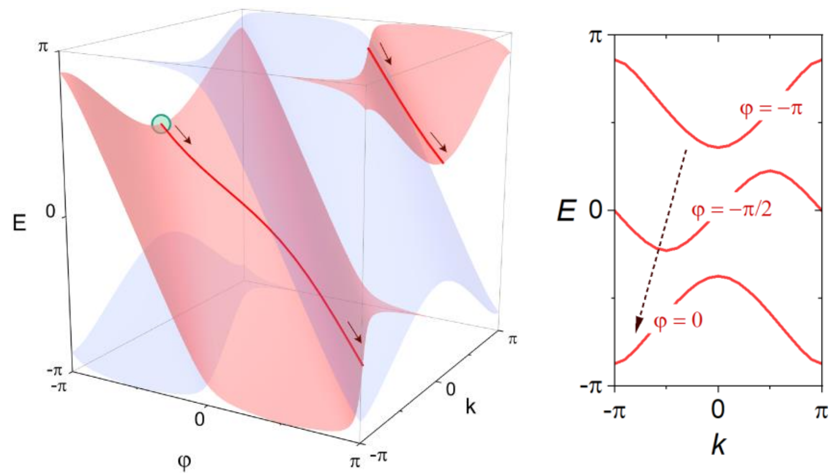
Theory

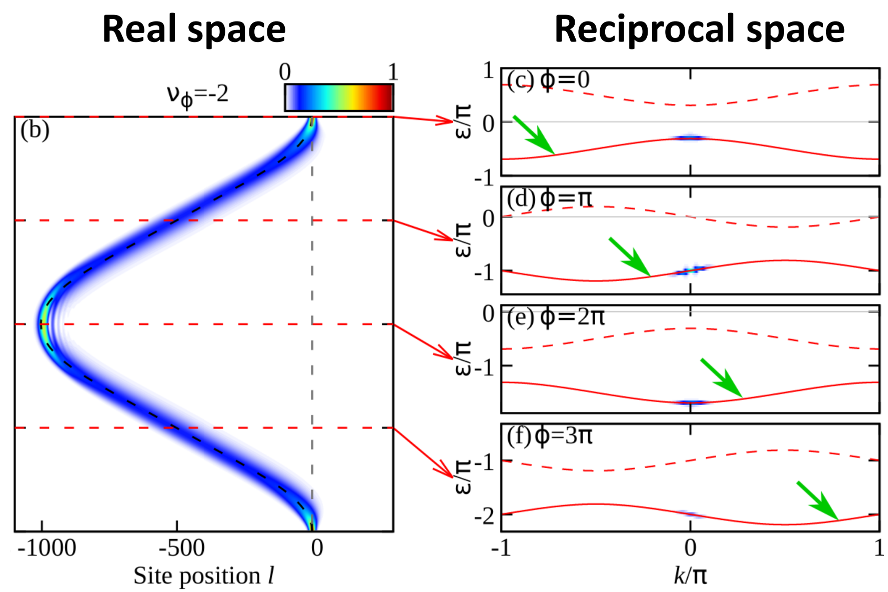


K is a topological invariant:

$$\nu = \frac{1}{2\pi i}\int_0^{2\pi} d\varphi \,\mathrm{Tr}\left[U_F^{-1}\frac{\partial U_F}{\partial \varphi}\right] = \sum_{j=\pm}\frac{1}{2\pi}\int_0^{2\pi} d\varphi \frac{\partial E_j}{\partial \varphi} = 2K$$

Topological Bloch sub-oscillaitons

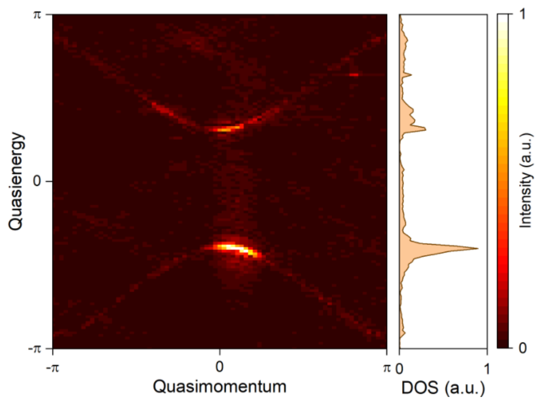


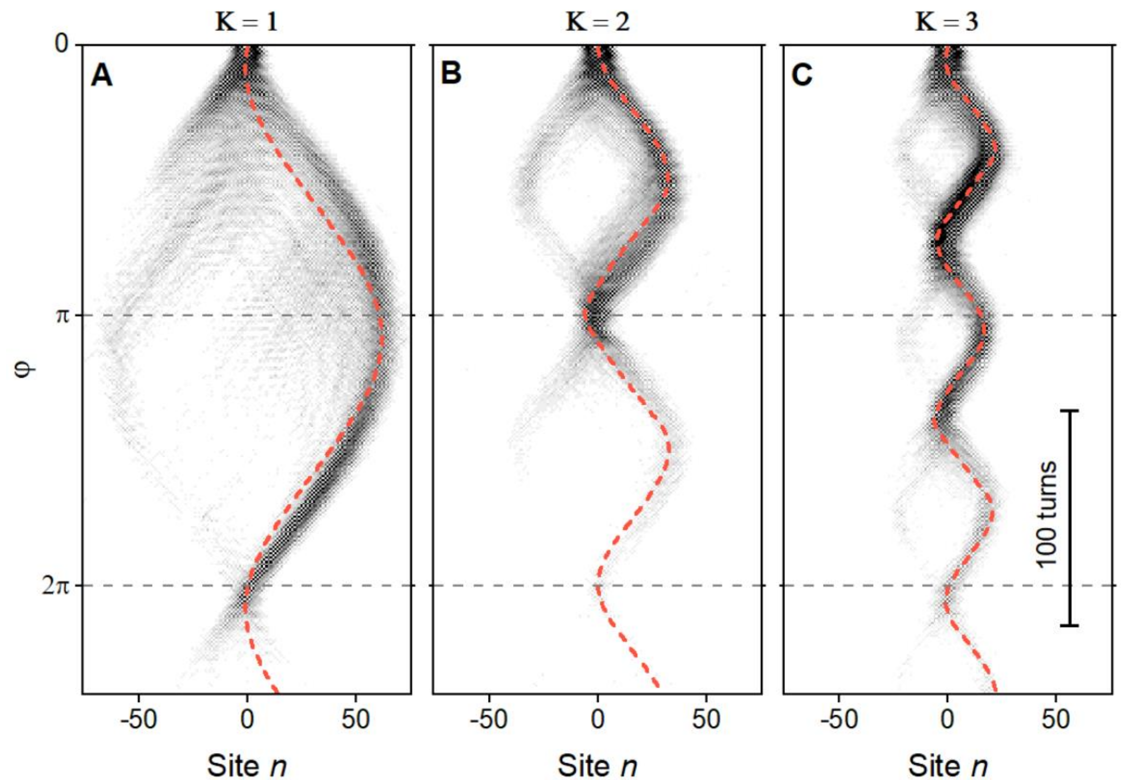


Upreti et al., Phys. Rev. Lett. 2020

Topological Bloch sub-oscillaitons

Exciting a single band:





Group velocity:

$$v_g^{\pm}(k,\varphi) = \frac{\partial E_{\pm}(k,\varphi)}{\partial k} = \pm \frac{\cos\theta_1 \cos\theta_2 \sin(k + K\varphi)}{\sqrt{1 - \left[\cos\theta_1 \cos\theta_2 \cos(k + K\varphi) - \sin\theta_1 \sin\theta_2 \cos(\Delta\varphi)\right]^2}}$$

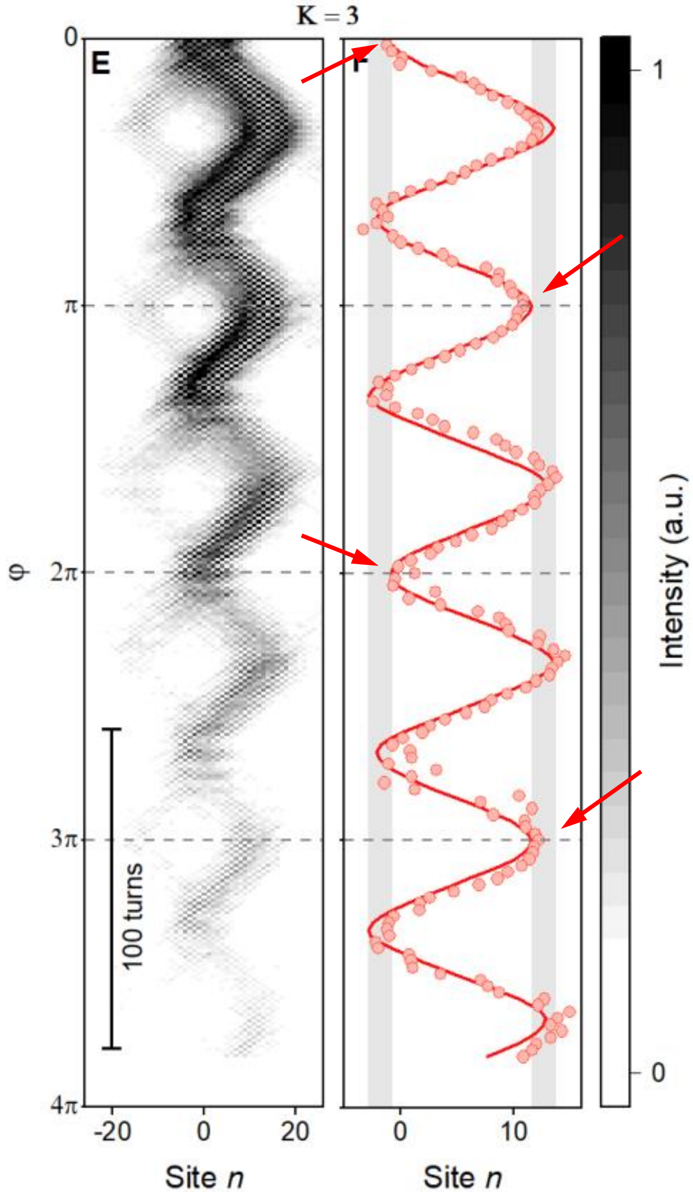
Frequency of sub-oscillations is topologically protected

Topological Bloch sub-oscillaitons

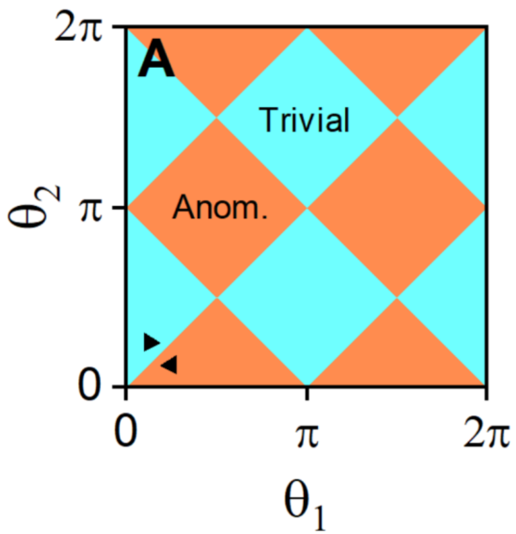
Amplitude of sub-oscillations can change!

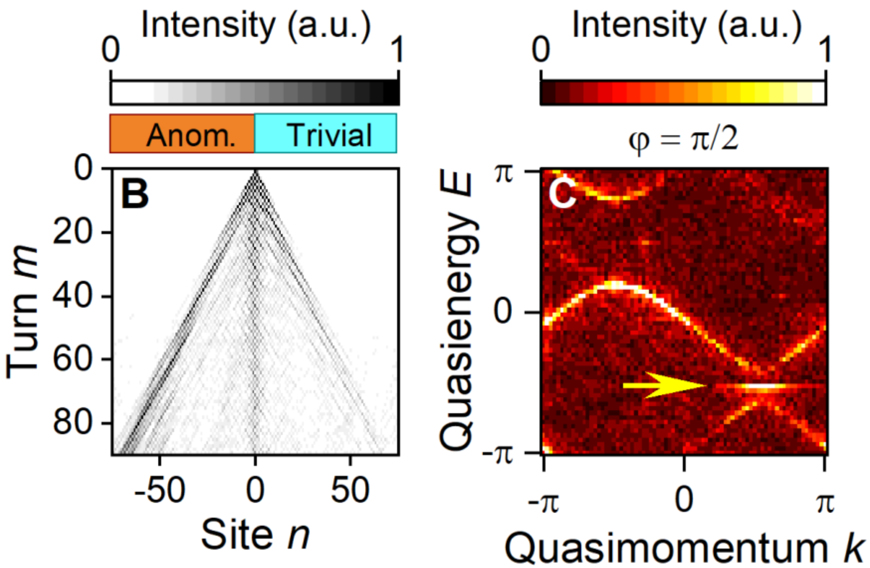
Group velocity:

$$\pm \frac{\cos\theta_1 \cos\theta_2 \sin(k + K\varphi)}{\sqrt{1 - [\cos\theta_1 \cos\theta_2 \cos(k + K\varphi) - \sin\theta_1 \sin\theta_2 \cos(\Delta\varphi)]^2}}$$

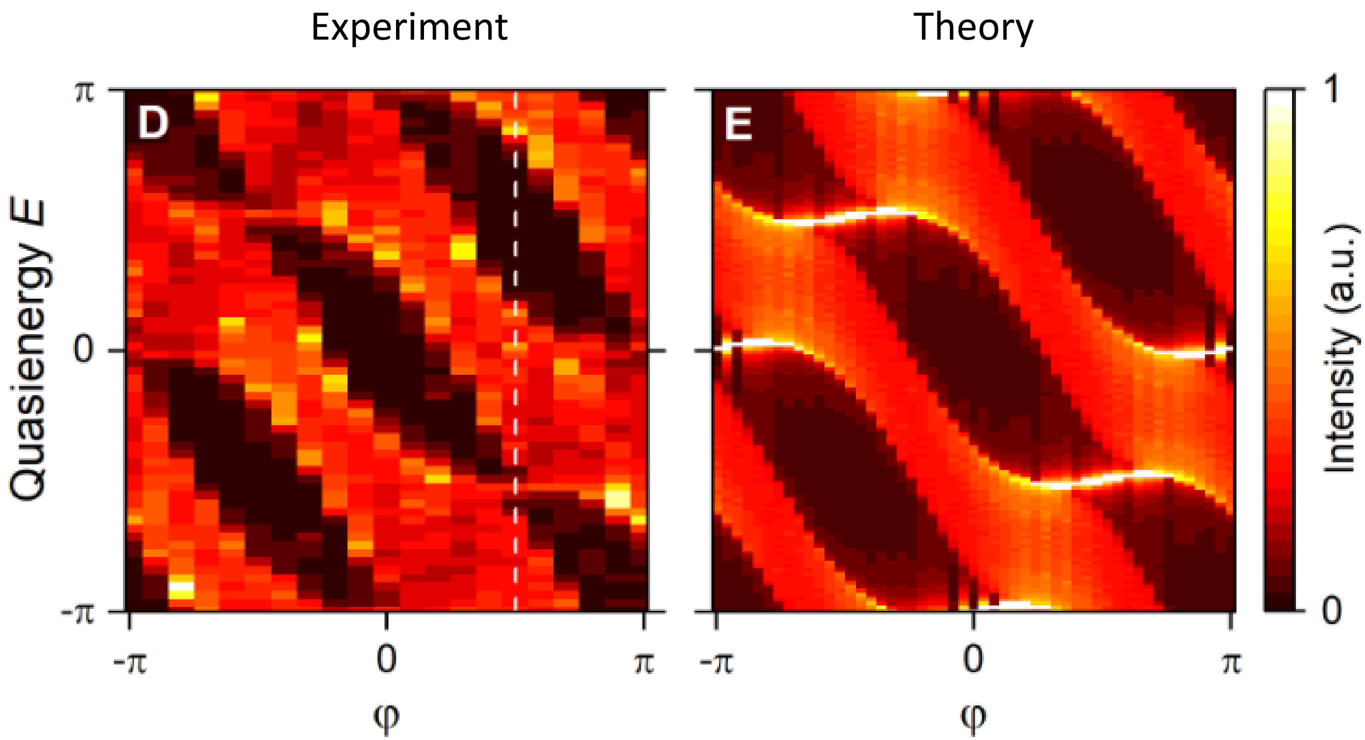


Phase diagram



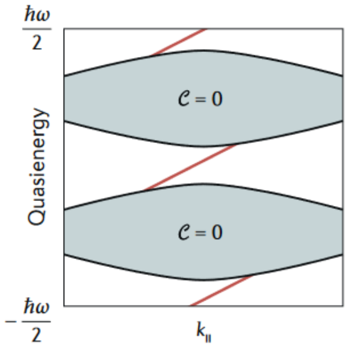


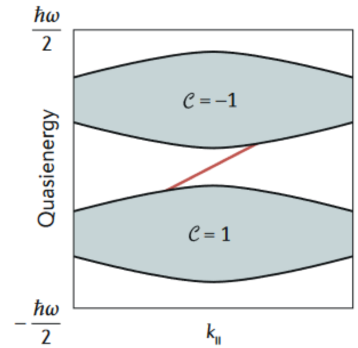
2D: Edge states + winding



Two topological invariants coexist!

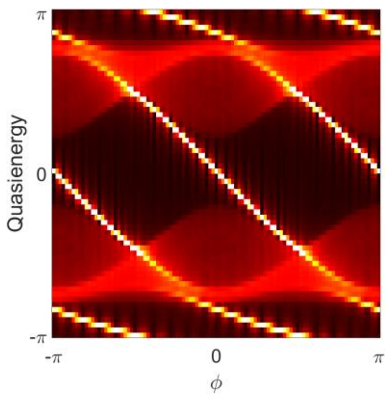
Next steps: Chern number, Berry curvature





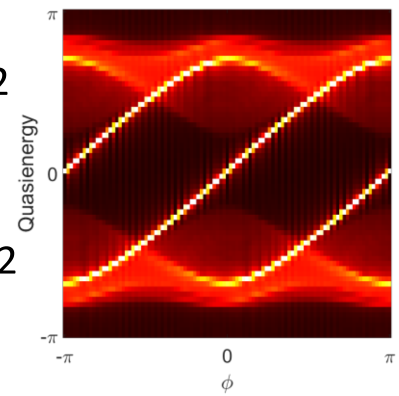
C = 0

C = 0



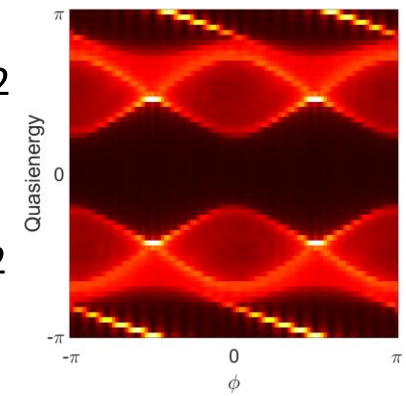
C = -2

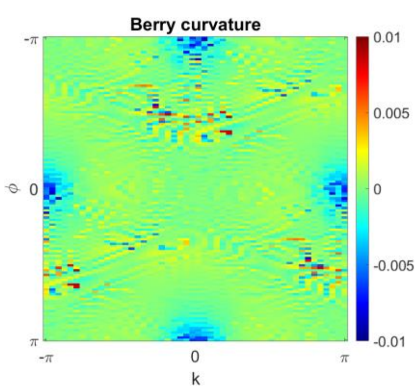
C = +2

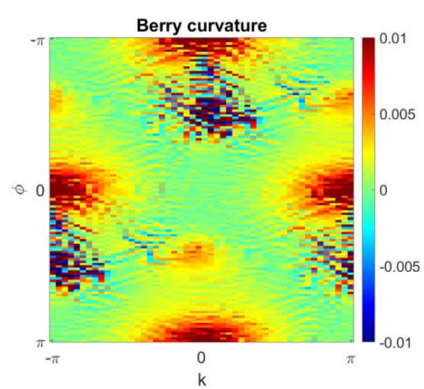


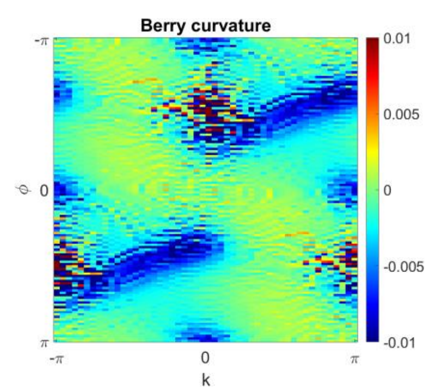
C = +2

C = -2



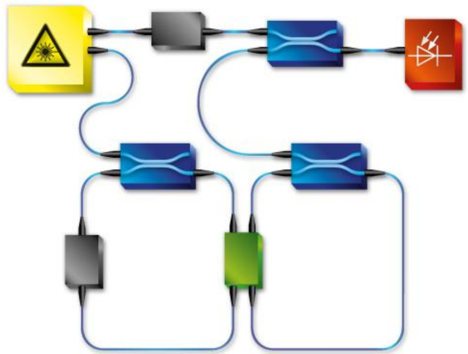






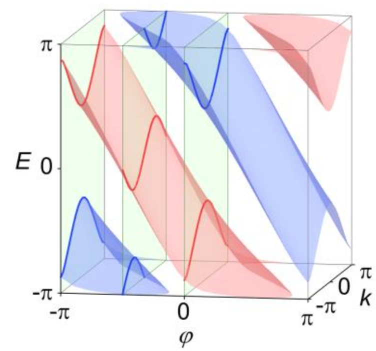
Summary and outlook

Universal photonic simulator for studying topological effects



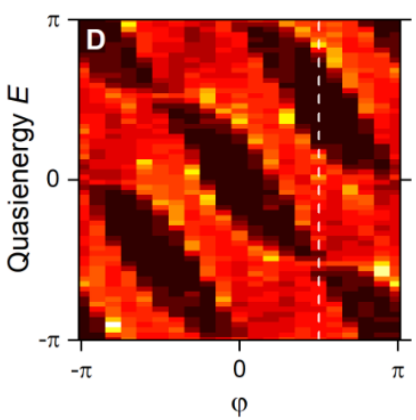
Lechevalier et al., Commun. Phys. 2021

Realization of a Floquet winding metal

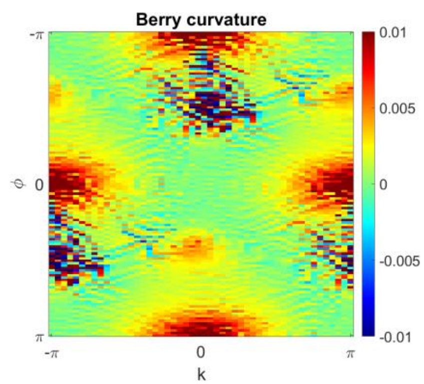


Upreti et al., Phys. Rev. Lett. 2020
Adiyatullin et al., arXiv:2203:01056

Coexistence of two topological invariants

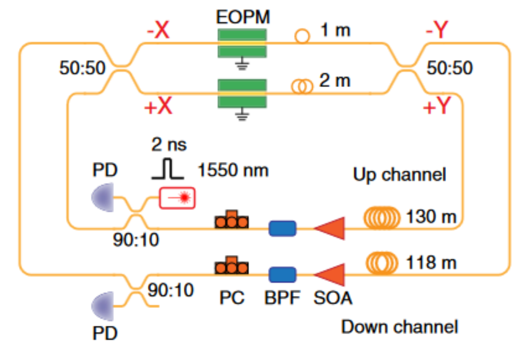


Measuring the invariants



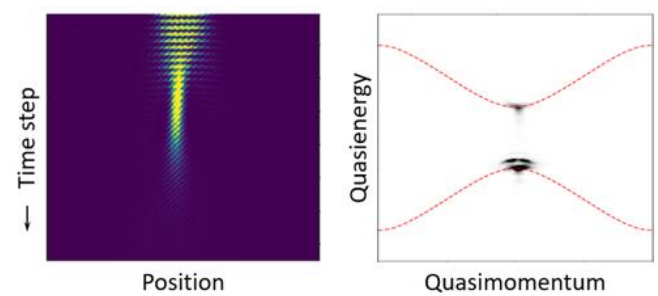
Next steps

Expanding to higher dimensions



Chalabi et al., Phys. Rev. Lett. 123, 150503 (2019)

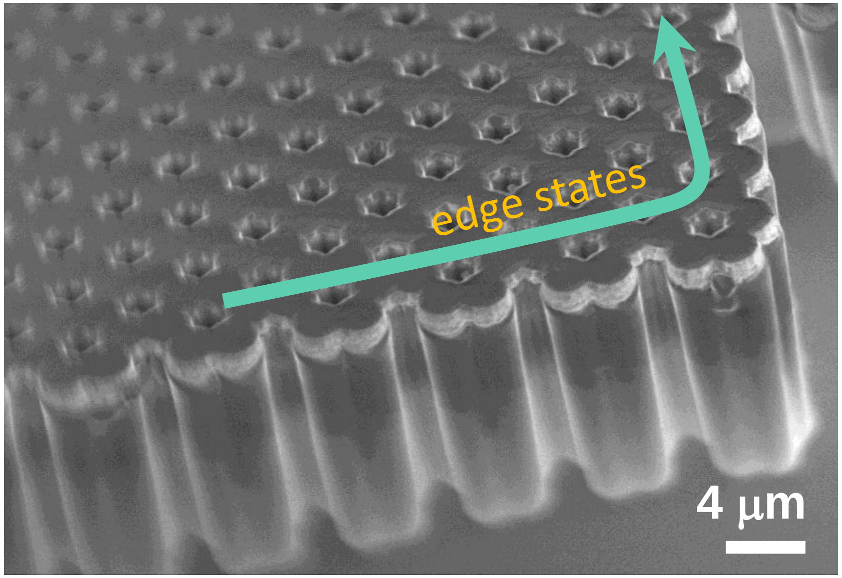
Interactions due to fiber nonlinearities



by C. Lechevalier

Topology in exciton-polariton lattices

2D lattice of polariton resonators



O. Jamadi et al., Light Sci. Appl. 9, 144 (2020)